Shear fields in fluids


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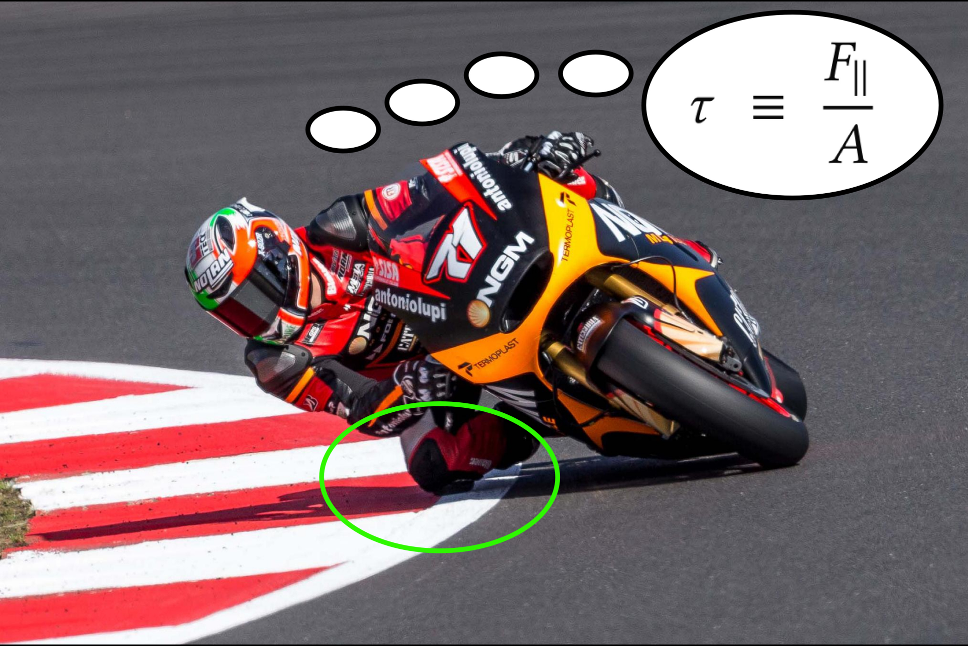


$\tau \equiv \frac{F_{\parallel}}{A}$

Back in chapter 1:

$$\tau \equiv \frac{F_{\|}}{A}$$

We need to move beyond this!

The direction of shear

- Which direction is “parallel to the plate”?
- And which plate anyway?

There Is No Plate

$$\vec{\tau} \equiv \lim_{A \to 0} \frac{\vec{F}_{\parallel}}{A}$$

Shear is a *field*

Shear at a given point has three components:

$$\vec{\tau} \;=\; \left(\tau_x, \tau_y, \tau_z\right)$$

In a fluid, there is a shear *vector field*:

$$\vec{\tau}_{(x,y,z,t)} \;\equiv\; \begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}_{(x,y,z,t)}$$

Shear on an infinitesimal volume

~ things are getting interesting ~

aha!

No more plate: imagine a *cube*

Six different shears
each of three components

⟶ We need another dimension

woo!

$$\vec{\tau}_{\text{volume}} = \begin{pmatrix} \vec{\tau}_1 \\ \vec{\tau}_2 \\ \vec{\tau}_3 \\ \vec{\tau}_4 \\ \vec{\tau}_5 \\ \vec{\tau}_6 \end{pmatrix} = \begin{pmatrix} \tau_{1x}, \tau_{1y}, \tau_{1z} \\ \tau_{2x}, \tau_{2y}, \tau_{2z} \\ \tau_{3x}, \tau_{3y}, \tau_{3z} \\ \tau_{4x}, \tau_{4y}, \tau_{4z} \\ \tau_{5x}, \tau_{5y}, \tau_{5z} \\ \tau_{6x}, \tau_{6y}, \tau_{6z} \end{pmatrix}$$

eighteen ways to scratch a cat

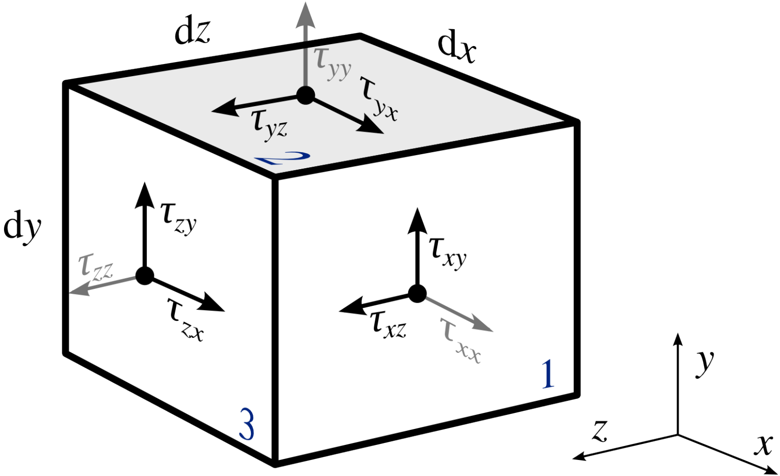
Writing it the smart way

$$\tau_{ab}$$

First subscript (a): direction perpendicular to plane
Second subscript (b): direction of shear

So, on face perpendicular to x

$$\begin{aligned}\vec{\tau}_{xj} &= \vec{\tau}_{xx} + \vec{\tau}_{xy} + \vec{\tau}_{xz} \\ &= \tau_{xx}\vec{i} + \tau_{xy}\vec{j} + \tau_{xz}\vec{k}\end{aligned}$$



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The shear tensor

Formal notation

$$\vec{\tau}_{ij} \equiv \begin{pmatrix} \vec{\tau}_{xj} \\ \vec{\tau}_{yj} \\ \vec{\tau}_{zj} \end{pmatrix} \equiv \begin{pmatrix} \vec{\tau}_{xj\ \{1,4\}} \\ \vec{\tau}_{yj\ \{2,5\}} \\ \vec{\tau}_{zj\ \{3,6\}} \end{pmatrix}$$

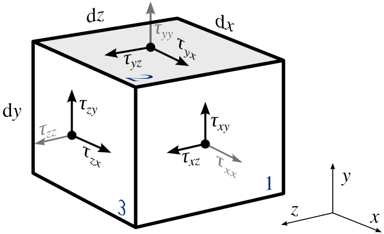
$$= \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

nine (pairs of) components shear tensor

Shear has two effects on particles:

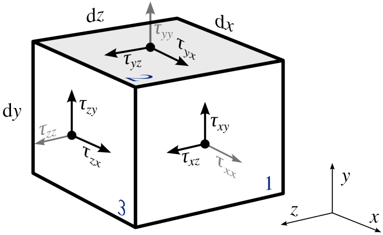
1. it deforms (strains) them;
2. it accelerates them (changes their $\vec{V}$).

What is the net *force* due to shear on the particle?



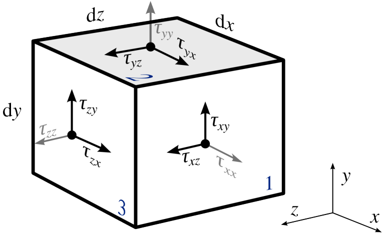
$$\vec{F}_{\text{shear } x} = S_3 \vec{\tau}_{zx\ 3} - S_6 \vec{\tau}_{zx\ 6} + S_2 \vec{\tau}_{yx\ 2} - S_5 \vec{\tau}_{yx\ 5} + S_1 \vec{\tau}_{xx\ 1} - S_4 \vec{\tau}_{xx\ 4}$$

What is the net *force* due to shear on the particle?



$$\begin{aligned}\vec{F}_{\text{shear } x} = \quad & S_3 \vec{\tau}_{zx\ 3} - S_3 \vec{\tau}_{zx\ 6} \\ & + S_2 \vec{\tau}_{yx\ 2} - S_2 \vec{\tau}_{yx\ 5} \\ & + S_1 \vec{\tau}_{xx\ 1} - S_1 \vec{\tau}_{xx\ 4}\end{aligned}$$

What is the net *force* due to shear on the particle?



$$\begin{aligned}\vec{F}_{\text{shear } x} = \quad & \mathrm{d}x\,\mathrm{d}y(\vec{\tau}_{zx\ 3} - \vec{\tau}_{zx\ 6}) \\ & + \mathrm{d}x\,\mathrm{d}z(\vec{\tau}_{yx\ 2} - \vec{\tau}_{yx\ 5}) \\ & + \mathrm{d}z\,\mathrm{d}y(\vec{\tau}_{xx\ 1} - \vec{\tau}_{xx\ 4})\end{aligned}$$

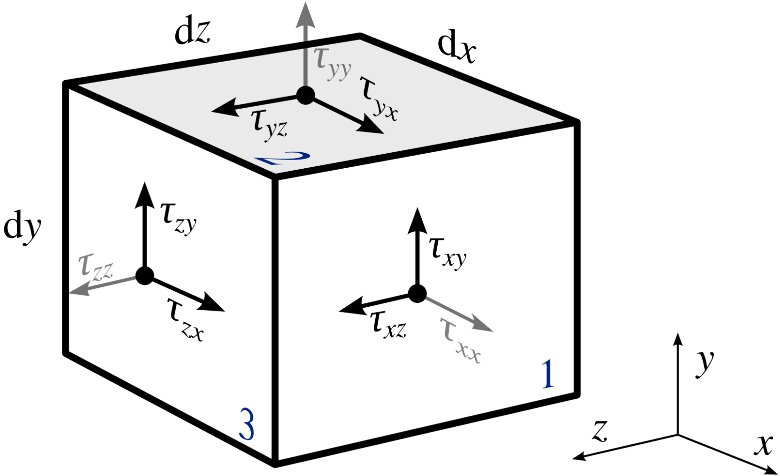
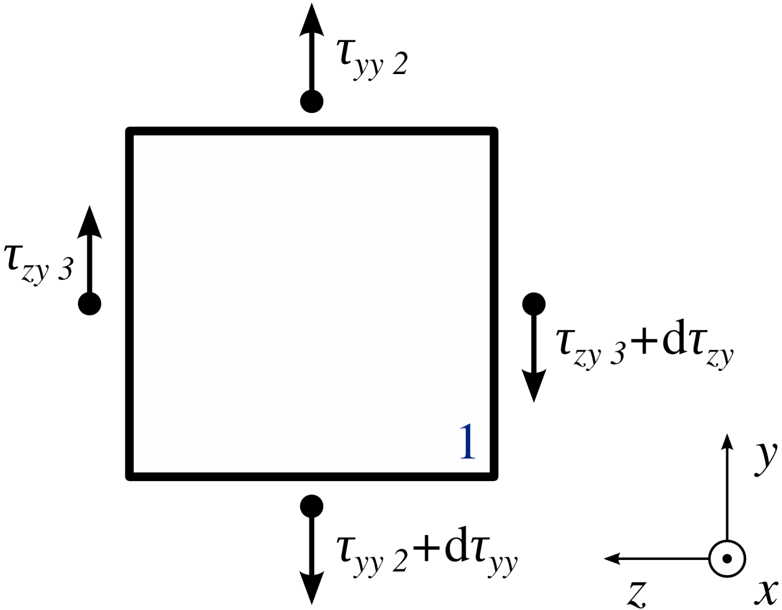


Figure CC-0 Olivier Cleynen



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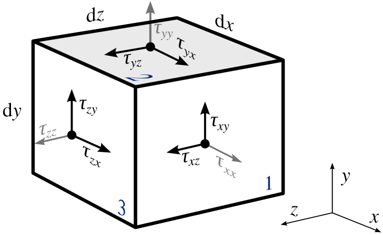
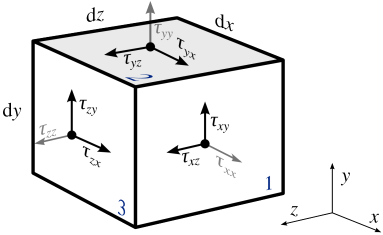
What is the net *force* due to shear on the particle?



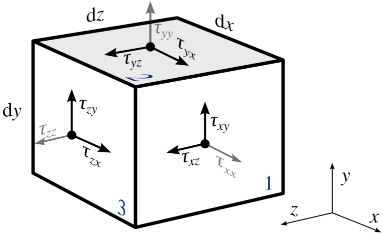
$$
\begin{aligned}
\vec{F}_{\text{shear } x} = \quad & \mathrm{d}x\,\mathrm{d}y(\vec{\tau}_{zx\ 3} - \vec{\tau}_{zx\ 6}) \\
& + \mathrm{d}x\,\mathrm{d}z(\vec{\tau}_{yx\ 2} - \vec{\tau}_{yx\ 5}) \\
& + \mathrm{d}z\,\mathrm{d}y(\vec{\tau}_{xx\ 1} - \vec{\tau}_{xx\ 4})
\end{aligned}
$$

What is the net *force* due to shear on the particle?



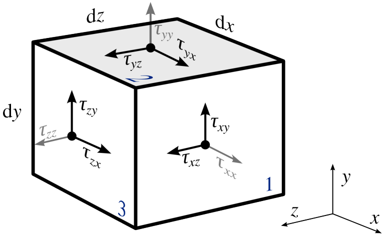
$$\vec{F}_{\text{shear } x} = \mathrm{d}x\,\mathrm{d}y\,(\mathrm{d}\vec{\tau}_{zx}) + \mathrm{d}x\,\mathrm{d}z\,(\mathrm{d}\vec{\tau}_{yx}) + \mathrm{d}z\,\mathrm{d}y\,(\mathrm{d}\vec{\tau}_{xx})$$

What is the net *force* due to shear on the particle?



$$\vec{F}_{\text{shear } x} = \mathrm{d}x\,\mathrm{d}y\left(\mathrm{d}z\frac{\partial\vec{\tau}_{zx}}{\partial z}\right) + \mathrm{d}x\,\mathrm{d}z\left(\mathrm{d}y\frac{\partial\vec{\tau}_{yx}}{\partial y}\right) + \mathrm{d}z\,\mathrm{d}y\left(\mathrm{d}x\frac{\partial\vec{\tau}_{xx}}{\partial x}\right)$$

What is the net *force* due to shear on the particle?



$$\vec{F}_{\text{shear } x} = \mathrm{d}\mathcal{V}\left(\frac{\partial \vec{\tau}_{zx}}{\partial z} + \frac{\partial \vec{\tau}_{yx}}{\partial y} + \frac{\partial \vec{\tau}_{xx}}{\partial x}\right)$$

there must be a better way to write this!