Exercise sheet 2 (Effects of shear)

last edited April 30, 2017

These lecture notes are based on textbooks by White [13], Çengel & al.[16], and Munson & al.[18].

Except otherwise indicated, we assume that fluids are Newtonian, and that:
$\rho_{\text{water}} = 1\,000\,\text{kg}\,\text{m}^{-3}$; $p_{\text{atm.}} = 1\,\text{bar}$; $\rho_{\text{atm.}} = 1{,}225\,\text{kg}\,\text{m}^{-3}$; $T_{\text{atm.}} = 11{,}3\,°\text{C}$; $\mu_{\text{atm.}} = 1{,}5 \cdot 10^{-5}\,\text{N}\,\text{s}\,\text{m}^{-2}$;
$g = 9{,}81\,\text{m}\,\text{s}^{-2}$. Air is modeled as a perfect gas ($R_{\text{air}} = 287\,\text{J}\,\text{K}^{-1}\,\text{kg}^{-1}$; $\gamma_{\text{air}} = 1{,}4$; $c_{p\text{air}} = 1\,005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$).

2.1 Flow in between two plates

Munson & al. [18] Ex1.5

A fluid is forced to flow between two stationary plates (fig. 2.4). We observe that the flow is laminar (smooth and fully steady), with a univorm velocity profile $u = f(y)$ which is linked to the average fluid velocity V_{average} by the relationship:

$$u = \frac{3}{2} V_{\text{average}} \left[1 - \left(\frac{y}{H} \right)^2 \right] \tag{2/25}$$

where y is measured from the middle of the gap;
and H is half of the gap length.

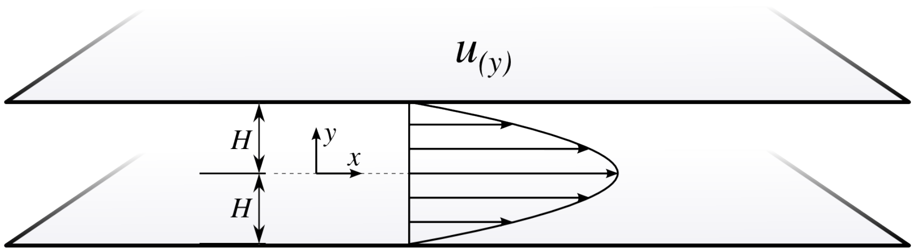

Figure 2.4 – Velocity distribution for laminar flow in between two plates, also known as *Couette flow*.

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The fluid viscosity is $2\,\text{N}\,\text{s}\,\text{m}^{-2}$, the average velocity is $0{,}6\,\text{m}\,\text{s}^{-1}$ and the two plates are $10\,\text{mm}$ apart.

1. What is the shear effort $\tau_{yx\ \text{plate}}$ generated on the lower plate?
2. What is the shear effort τ_{yx} in the middle plane of the flow?

2.2 Friction on a plate

A plate the size of an A4 sheet of paper (210 mm × 297 mm) is moved horizontally at constant speed above a large flat surface (fig. 2.5). We assume that the velocity profile of the fluid betweeen the plate and the flat surface is entirely uniform, smooth, and steady.

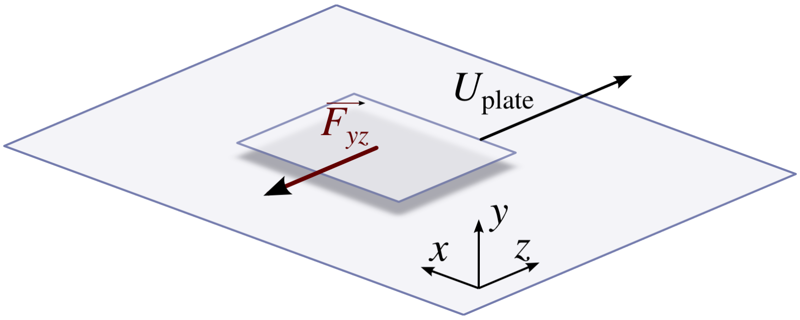

Figure 2.5 – A plate moved horizontally across a flat surface.

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1. Express the force F_{yz} due to shear on the plate as a function of its velocity U_{plate}, the gap height H, and the properties of the fluid.
2. The plate speed is $U_{plate} = 1\,\mathrm{m\,s^{-1}}$ and the gap height is $H = 5\,\mathrm{mm}$. What is the shear force if the fluid is air ($\mu_{atm.} = 1{,}5 \cdot 10^{-5}\,\mathrm{N\,s\,m^{-2}}$), and if the fluid is honey ($\mu_{honey} = 40\,\mathrm{N\,s\,m^{-2}}$)?
3. If a very long and thin plate with the same surface area was used instead of the A4-shaped plate, would the shear force be different? (briefly justify your answer, e.g. in 30 words or less)

2.3 Viscometer

Çengel & al. [16] 2-78

An instrument designed to measure the viscosity of fluids (named *viscometer*) is made of two coaxial cylinders (fig. 2.6). The inner cylinder is immersed in a liquid, and it rotates within the stationary outer cylinder.

The two cylinders are 75 cm tall. The inner cylinder diameter is 15 cm and the spacing is 1 mm.

When the inner cylinder is rotated at 300 rpm, a friction-generated moment of 0,8 N m is measured.

1. If the flow in between the cylinders corresponds to the simplest possible flow case (steady, uniform, fully-laminar), what is the viscosity of the fluid?
2. Would a non-Newtonian fluid induce a higher moment? (briefly justify your answer, e.g. in 30 words or less)

[Note: in practice, when the inner cylinder is turned at high speed, the flow displays mesmerizing patterns called Taylor—Couette vortices, the description of which is much more complex!]

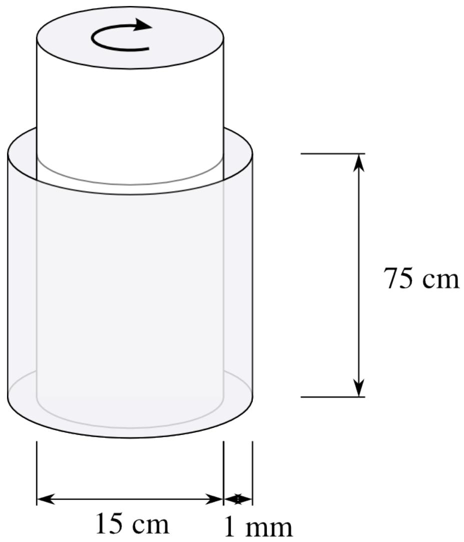

Figure 2.6 – Sketch of a cylinder viscometer. The width of the gap has been greatly exaggerated for clarity.

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2.4 Boundary layer

White [13] P1.56

A laminar fluid flow occurs along a wall (fig. 2.7). Close to the wall ($y < \delta$), we observe that viscous effects dominate the mechanics of the flow. This zone is designated *boundary layer*. The speed $u_{(y)}$ can then be modeled with the relation:

$$u = U \sin\left(\frac{\pi y}{2\delta}\right) \tag{2/26}$$

in which U is the flow speed far away from the wall.

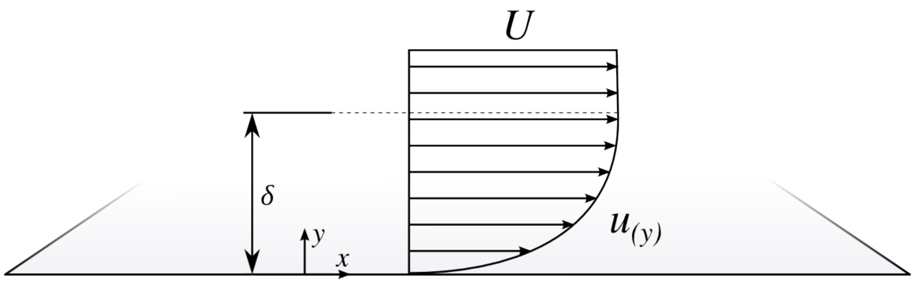

Figure 2.7 – Velocity profile across the boundary layer.

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The fluid is helium at 20 °C and 1 bar ($9 \cdot 10^{-6}$ N s m^{-2}); measurements yield $U = 10{,}8$ m s^{-1} and $\delta = 3$ cm.

1. What is the shear effort τ_{yx} on the wall?
2. At which height y_1 above the surface will the shear effort be half of this value?
3. What would be the wall shear if helium was replaced with water ($1 \cdot 10^{-3}$ kg m^{-1} s^{-1})?

2.5 Clutch

Çengel & al. [16] *2-74*

Two aligned metal shafts are linked by a clutch, which is made of two disks very close one to another, rotating in the same direction at similar (but not identical) speeds. The disk diameters are both 30 cm and the gap between them is 2 mm ; they are submerged in SAE30W oil with viscosity 0,38 $kg\,m^{-1}\,s^{-1}$.

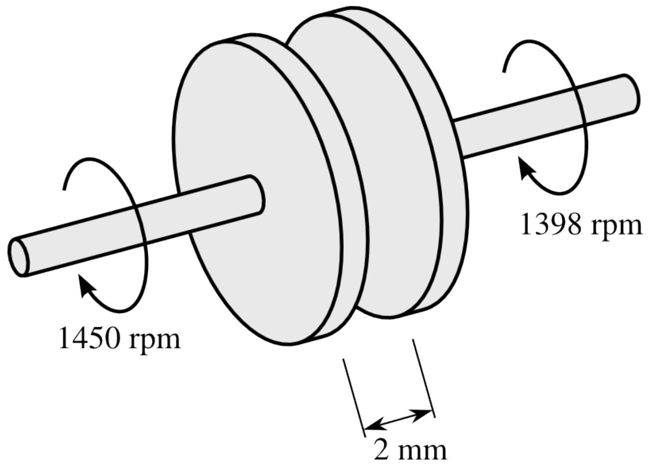

Figure 2.8 – Sketch of the two disks constituting the clutch. The gap width has been exaggerated for clarity.

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The power shaft rotates at 1 450 rpm, while the powered shaft rotates at 1 398 rpm. We consider the simplest possible flow case (steady, laminar) in between the two disks.

1. What is the moment imparted by one disk to the other?
2. How would the moment change if the radius of each disk was doubled?
3. What is the transmitted power and the clutch efficiency?
4. Briefly (e.g. in 30 words or less) propose one reason why in practice the flow in between the two disks may be different from the simplest-case flow used in this exercise.

Answers

2.1 1) $\tau_{yx}|_{y=-H} = 720\,\mathrm{N\,m^{-2}}$; 2) $\tau_{yx}|_{y=0} = 0\,\mathrm{N\,m^{-2}}$.

2.2 1) $F_{yz} = L_1 L_2 \mu \frac{U}{H} = 1{,}87 \cdot 10^{-4}\,\mathrm{N}$ for air, and 500 N for honey(!).

2.3 $\mu = \frac{hM}{2\pi\omega R_1^3 H} = 1{,}281 \cdot 10^{-2}\,\mathrm{N\,s\,m^{-2}}$.

2.4 1) $\tau_{yx,\mathrm{wall}} = \mu \frac{\pi U}{2\delta} = 5{,}09 \cdot 10^{-3}\,\mathrm{N^2\,m^{-1}}$ for helium;
2) $y_1 = \frac{2}{3}\delta = 2\,\mathrm{cm}$; 3) $\tau_{yx,\mathrm{wall}} = 0{,}565\,\mathrm{N\,m^{-2}}$ for water.

2.5 1) $M = \frac{\pi}{2}\frac{\mu\omega}{h}R^4 = 0{,}8228\,\mathrm{N\,m}$; 3) $\dot{W}_2 = \omega_2 M = 120\,\mathrm{W}$; $\eta_{\mathrm{clutch}} = \frac{\dot{W}_2}{\dot{W}_1} = 96{,}4\,\%$ (remember this is a very low-power, low-relative speed, laminar-flow case).