Exercise sheet 0 (Important concepts)

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Except otherwise indicated, we assume that fluids are Newtonian, and that:
$\rho_{water} = 1\,000\,\mathrm{kg\,m^{-3}}$; $p_{atm.} = 1\,\mathrm{bar}$; $\rho_{atm.} = 1{,}225\,\mathrm{kg\,m^{-3}}$; $T_{atm.} = 11{,}3\,°\mathrm{C}$; $\mu_{atm.} = 1{,}5 \cdot 10^{-5}\,\mathrm{N\,s\,m^{-2}}$;
$g = 9{,}81\,\mathrm{m\,s^{-2}}$. Air is modeled as a perfect gas ($R_{air} = 287\,\mathrm{J\,K^{-1}\,kg^{-1}}$; $\gamma_{air} = 1{,}4$; $c_{p\,air} = 1\,005\,\mathrm{J\,kg^{-1}\,K^{-1}}$).

0.1 Compressibility effects

An aircraft is flying in air with density $0{,}9\,\mathrm{kg\,m^{-3}}$ and temperature $-5\,°\mathrm{C}$. Above which flight speed would you expect the air flow over the wings to become compressible?

0.2 Pressure-induced force

A $2\,\mathrm{m}$ by $2\,\mathrm{m}$ flat panel is used as the wall of a swimming pool (fig. 0.3). On the left side, the pressure is uniform at $1\,\mathrm{bar}$.

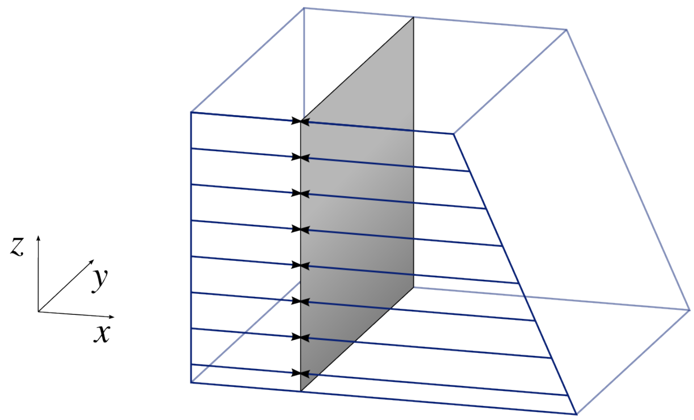

Figure 0.3 – Pressure distribution on a flat plate

Figure CC-0 o.c.

1. What is the pressure force exerted on the left side of the plate?

On the right side of the plate, the water exerts a pressure which is not uniform: it increases with depth. The relation, expressed in pascals, is:

$$p_{water} = 1{,}3 \cdot 10^{5} - 9{,}81 \cdot 10^{3} \times z \qquad (0/28)$$

2. What is the pressure force exerted on the right side of the plate?
   [Hint: we will explore the required expression in chapter 1 as eq. 1/14 p.35]

0.3 Shear-induced force

A fluid flows over a 3 m by 3 m flat horizontal plate, in the x-direction as shown in fig. 0.4. Because of this flow, the plate is subjected to uniform shear $\tau_{zx} = 1{,}65\,\text{Pa}$.

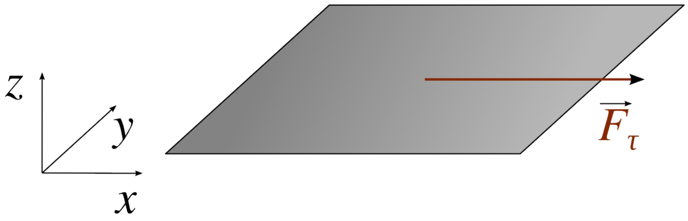

Figure 0.4 – Shear force exerting on a plate

Figure CC-0 o.c.

1. What is the shear force applying on the plate?
2. What would be the shear force if the shear was not uniform, but instead was a function of x expressed (in pascals) as $\tau_{zx} = 1{,}65 - 0{,}01 \times x^2$?
   [Hint: we will explore the required expression in chapter 2 as eq. 2/20 p.53]

0.4 Speed of sound

White [13] P1.87

Isaac Newton measured the speed of sound by timing the interval between observing smoke produced by a cannon blast and the hearing of the detonation. The cannon is shot 8,4 km away from Newton. What is the air temperature if the measured interval is 24,2 s? What is the temperature if the interval is 25,1 s?

0.5 Power lost to drag

A truck moves with constant speed $\vec{V}$ on a road, with $V = 100\,\text{km}\,\text{h}^{-1}$. Because it experiences cross-wind, it is subjected to a drag $\vec{F}_D$ with $F_D = 5\,\text{kN}$ at an angle $\theta = 20°$, as shown in fig. 0.5.

1. What is the power required to overcome drag?

The drag force $\vec{F}_D$ is applying at a distance 0,8 m behind the center of gravity of the truck.

2. What are the magnitude and the direction of the moment exerted by the drag $\vec{F}_D$ about the center of gravity?

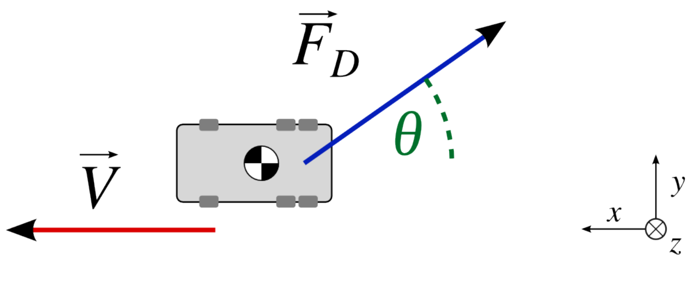

Figure 0.5 – Top view of a truck traveling at velocity $\vec{V}$ and subject to a drag force $\vec{F}_D$

Figure CC-0 o.c.

0.6 Go-faster exhaust pipe

The engine exhaust gases of a student's hot-rod car are flowing quasi-steadily in a cylindrical outlet pipe, whose outlet is slanted at an angle $\theta = 25°$ to improve the good looks of the car and provide the opportunity for an exercise.

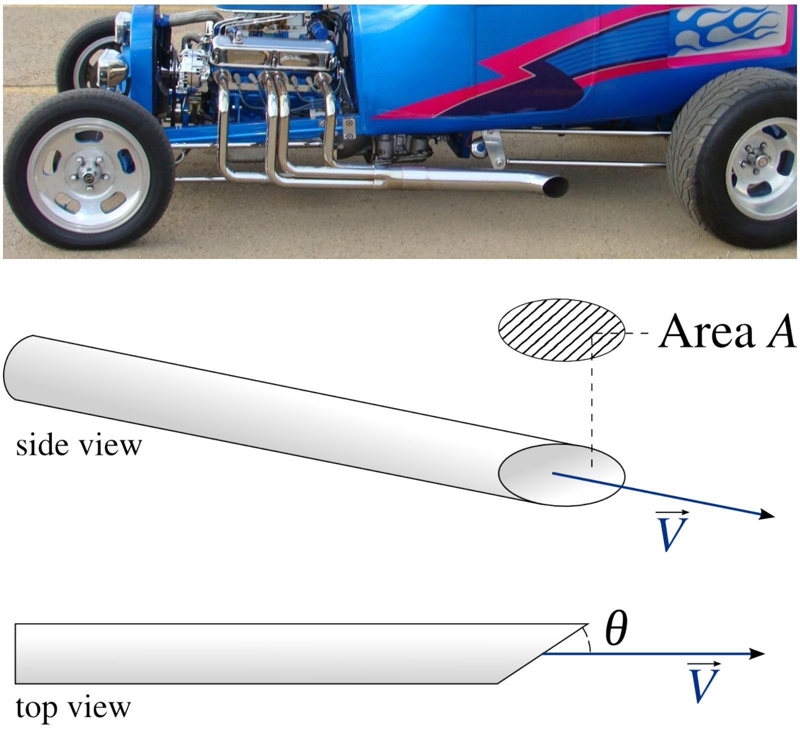

Figure 0.6 – Exhaust gas pipe of a car. The outlet cross-section is at an angle θ relative to the axis of the pipe.

The outlet velocity is measured at $15\,\mathrm{m\,s^{-1}}$, and the exhaust gas density is $1{,}1\,\mathrm{kg\,m^{-3}}$. The slanted outlet section area A is $420\,\mathrm{cm^2}$.

1. What is the mass flow $\dot{m}$ through the pipe?
2. What is the volume flow $\dot{\mathcal{V}}$ of exhaust gases?

Because of the shear within the exhaust gases, the flow through the pipe induces a pressure loss of $21\,\mathrm{Pa}$ (we will learn to quantify this in chapter 5). In these conditions, the specific heat capacity of the exhaust gases is $c_{p\,\mathrm{gases}} = 1\,100\,\mathrm{J\,kg^{-1}\,K^{-1}}$.

3. What is the power required to carry the exhaust gases through the pipe?
4. What is the gas temperature increase due to the shear in the flow?

0.7 Acceleration of a particle

Inside a complex, turbulent water flow, we are studying the trajectory of a cubic fluid particle of width $0{,}1\,\mathrm{mm}$. The particle is accelerating at a rate of $2{,}5\,\mathrm{m\,s^{-2}}$.

1. What is the net force applying to the particle?
2. In practice, which types of forces could cause it to accelerate?

0.8 Flow classifications

1. Can an incompressible flow also be unsteady?
2. Can a very viscous fluid flow in a turbulent manner?
3. *[more difficult]* Can a compressible flow also be isothermal?
4. Give an example of an isothermal flow, of an unsteady flow, of a compressible flow, and of an incompressible flow.

Answers

0.1 If you adopt [Ma] = 0,6 as an upper limit, you will obtain $V_{max} = 709\,\mathrm{km\,h^{-1}}$ (eqs. 0/10 & 0/11 p.16). Note that propellers, fan blades etc. will meet compressiblity effects far sooner.

0.2 1) $F_{left} = 400\,\mathrm{kN}$ (eq. 0/15 p.17); 2) $F_{right} = 480\,\mathrm{kN}$ (eq. 1/14 p.35).

0.3 1) $F_1 = 14{,}85\,\mathrm{N}$ (eq. 0/16 p.17); 2) $F_2 = 14{,}58\,\mathrm{N}$ (eq. 2/20 p.53).

0.4 26,7 °C & 5,6 °C.

0.5 1) $\dot{W} = \vec{F}_{drag} \cdot \vec{V}_{truck} = 130{,}5\,\mathrm{kW}$;

2) $M = ||\vec{r} \wedge \vec{F}_{drag}|| = 1\,368\,\mathrm{N\,m}$, $\vec{M} = \begin{pmatrix} 0 \\ 0 \\ -1\,368 \end{pmatrix}$ (points vertically upwards).

0.6 1) $\dot{m} = 0{,}2929\,\mathrm{kg\,s^{-1}}$ (eq. 0/17 p.18); 2) $\dot{\mathcal{V}} = 266{,}2\,\mathrm{L\,s^{-1}}$ (eq. 0/19 p.18); 3) $\dot{W} = 5{,}59\,\mathrm{W}$ (eq. 0/20 p.18); 4) $\Delta T = +0{,}0174\,\mathrm{K}$ (eq. 0/21 p.18), an illustration of remarks made in §0.8 p.21 regarding temperature distribution.

0.7 1) $F_{net} = 2{,}5 \cdot 10^{-9}\,\mathrm{N}$ (eq 0/23 p.19), such are the orders of magnitude involved in CFD calculations! 2) Only three kinds: forces due pressure, shear, and gravity.

0.8 1) yes, 2) yes if [Re] is high enough, 3) yes (in very specific cases such as high pressure changes combined with high heat transfer or high irreversibility, therefore generally no), 4) open the cap of a water bottle and turn it upside down: you have an isothermal, unsteady, incompressible flow. An example of compressible flow could be the expansion in a jet engine nozzle.