Hello! You are consulting an examination paper from the archives at https://fluidmech.ninja/.

In the summer semester 2020, the general structure of the examination will be largely the same as in this archive. Nevertheless, because the course content progressively changes from year to year, there are a few differences. In former years,

• The course contained a chapter about compressible air flow (involving tables for air properties) that is no longer part of the course now;

• Conversely, several chapters have been added to the examinable content over the years;

• The course contained a duct flow problem involving a ball fountain (“Kugel fountain”) that is no longer part of the course now;

• Viscosity values were read in a different diagram, and may not match values read in the 2020 viscosity diagram;

• Many small updates in the notation had not yet been carried out.

To obtain precise information about the summer semester 2020 examination, consult the dedicated appendix in the lecture notes. If you have questions, contact me as detailed in the course syllabus. Thanks, and good luck in your revisions!

Olivier Cleynen
September 2020
Fluid Dynamics
Examination 2018-07-12
https://fluidmech.minja

1. Basic equations

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \rho \frac{\partial V_x}{\partial x} - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{(\partial x)^2} + \frac{\partial^2 u}{(\partial y)^2} + \frac{\partial^2 u}{(\partial z)^2} \right) \]

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \rho \frac{\partial V_y}{\partial y} - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{(\partial x)^2} + \frac{\partial^2 v}{(\partial y)^2} + \frac{\partial^2 v}{(\partial z)^2} \right) \]

\[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = \rho \frac{\partial V_z}{\partial z} - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{(\partial x)^2} + \frac{\partial^2 w}{(\partial y)^2} + \frac{\partial^2 w}{(\partial z)^2} \right) \]

1.2

Eq (5) applies without restrictions to all fluids, all the time.
(2) Piping and power of turbine

Diagram:

- Points: A, B, C, D, E, F, G
- Flow On and Off
- Δp_turbine

Distance

Diagram shows a pipeline with points A, B, C, D, E, F, and G, indicating the flow on and off with Δp_turbine.
\[ E = 6 \times 10^{-3} \text{ m} \]
\[ d = 1,2 \text{ m} \]
\[ q = 5 \text{ m}^3/\text{s} \rightarrow V_{av} = \frac{q}{A} = \frac{q}{\pi \frac{d^2}{4}} = \frac{5 \times 4}{\pi \times 1,2^2} \]
\[ V_{av} = 4,481 \text{ m/s} \]

\[ [Re]_D = \frac{\rho V_{av} D}{\mu} = \frac{10^3 \times 4,481 \times 1,2}{10^{-3}} \]

\[ [Re]_D = 5,305 \times 10^5 \]

From Moody diagram (with \( \frac{E}{D} \) and \([Re]_D\))

\[ f = 0,031 \]

\[ \Delta P = f \frac{L}{D} \frac{1}{2} \rho V_{av}^2 \]

\[ \Delta P = 0,031 \times \frac{0,8 \times 10^3}{1,2} \times 0,5 \times 10^3 \times 4,481^2 \]

\[ \Delta P = 8,0186 \times 10^5 \text{ Pa} = 8,0186 \text{ bar} \]
\[ \Delta P_{\text{diff}} = \rho \, g \, (\bar{h} + 2) = 10^3 \times 9.81 \times \left[ 5 - (22 + 25) \right] \]

\[ \Delta P_{\text{diff}} = -4.1202 \times 10^5 \, \text{Pa} = -4.1202 \, \text{bar} \]

\[ W_{\text{turbine}} = \dot{m} \left( \Delta P_{\text{diff}} + \Delta P_{\text{f}} \right) \]

\[ W_{\text{turbine}} = 5 \left( -4.1202 + 2.0036 \right) \times 10^5 \]

\[ W_{\text{lubric}} = -1.0503 \times 10^6 \, \text{W} = -1.0503 \, \text{kW} \]

(power lost by the water to the turbine)

2.4

With half the volume \( V_{\text{av}} \), \( \dot{V}_2 = 0.5 \, \dot{V}_1 \)

\[ V_{\text{av}2} = \frac{1}{2} \, V_{\text{av}1} \]

\[ \left[ \rho_2 \right]_{01} = \frac{1}{2} \left[ \rho_1 \right]_{01} = 8.653 \times 10^5 \rightarrow \text{friction factor is unchanged} \]

\[ \dot{W}_{\text{turbine}} = \frac{1}{4} \left| \Delta P_{\text{f}} \right|_{\text{turbine}} = 0.5048 \times 10^5 \, \text{Pa} = \left| \Delta P_{\text{f}} \right|_{\text{turbine}} \]
So the new power becomes

\[ \dot{w}_2 = \dot{v}_2 \left( \Delta p_{att} + \Delta \phi_k E \right) \]

\[ = \frac{1}{2} \dot{v}_1 \left( \Delta p_{att} + \frac{1}{4} \Delta \phi_k E \right) \]

\[ = 2.5 \left( -4.180 e + 0.5049 \right) \times 10^5 \]

\[ \dot{w} = -0.90238 \times 10^6 \dot{w} = -0.90238 \text{ MW} \]

\[ \frac{\dot{w}_2}{\dot{w}_1} = 86 \% : 14 \% \text{ decrease} \]
Viscometer

Overview:

On disk surface:
Flux $\tau$ on infinitesimal area $dA$ at radius $r$ is

$$d\tau = r \tau dA$$

Area is annulus of circumference $2\pi r$, thickness $dr$

$$d\tau = r \tau 2\pi r dr$$

Shear is due to velocity gradient above surface $dA$:

$$\tau = \mu \frac{du_\theta}{dz}$$

and $u_\theta$ is proportional to distance $z$ away from surface, and to radius $r$:

$$u_\theta = 0 + \frac{\omega r}{H} z$$

![Diagram showing $z$, $H$, $\omega r$, and $u_\theta$.]

$$d\tau = r \mu \frac{du_\theta}{dz} 2\pi r dr$$

$$= r \mu \frac{d}{dz} \left( \frac{\omega r}{H} z \right) 2\pi r dr = \frac{\mu \omega 2\pi}{H} r^3 dr$$
Now, integrating from \( r = 0 \) to \( r = R \):

\[
\eta = \int_0^R \frac{\mu w 2\pi r^3}{H} \, dr
\]

\[
\eta = \frac{\mu \pi w}{H} \times 2 \times \frac{1}{4} R^4
\]

and since \( R = \frac{D}{2} \)

\[
\eta = \frac{\mu \pi w D^4}{32 H}
\]

3.2

\[
\mu = \frac{32 H \eta}{\pi w D^4}
\]

\[
= \frac{32 \times 0.4 \times 10^{-3} \times 4.6 \times 10^{-3}}{\pi \times 8 \times \frac{2\pi}{60} \times (3 \times 10^{-2})^4}
\]

\[
\mu = 0.341 \text{ Pa s}
\]
Based on Fig. 1, the fluid is probably a thick liquid (oil, glycerin, etc). Those fluids see their viscosity decrease with increasing temperature.

So, it is likely that when $T^A$, $\mu \approx \varphi $

and $\Pi \approx \varphi$ based on eq. (2).
4. Pressure force on door

4.1

[Diagram showing forces and labels: inside, outside, Palm, Palm + P_water]

4.2

[Diagram showing variables: z, Z_{max}, P_{max}, dr, r]
\[ F_{net} = \int_{S} dF_{net} = \int_{S} P_{net} \, dA \]
\[ = \int_{S} \rho g z L_1 \, dr \]
\[ = \int_{0}^{R_{max}} \rho g z L_1 \, dr \]
\[ = \rho g L_1 \int_{0}^{R_{max}} z \, dr \]
\[ = \rho g L_1 \int_{0}^{R_{max}} (R_{max} - r \sin \theta) \, dr \]
\[ F_{net} = \rho g L_1 \left( R_{max}^2 - \frac{1}{2} \sin \theta \right) \left( R_{max} \right) \]
\[ = 10^3 \times 3.81 \times 3 \times \left[ 3.7 \frac{2.5}{\sin 60} - \frac{1}{2} \sin 60 \times \frac{815^2}{(\sin 60)^2} \right] \]
\[ = 10^3 \times 3.81 \times 3 \times \frac{2.5}{\sin 60} \left[ 3.7 - \frac{1}{2} \times 815 \right] \]
\[ = 2,081 \times 10^5 \text{N} = 208,1 \text{kN} = F_{net} \]
\[ \Pi_{\text{net}} = \int_{s} r \, dF_{\text{net}} = \rho g L_{1} \int_{0}^{R_{\text{max}}} 2 \, r \, dr \]

\[ \Pi_{\text{net}} = \rho g L_{1} \left[ \frac{1}{2} R_{\text{max}}^{2} - \frac{1}{3} \sin \theta \, R_{\text{max}}^{3} \right] \]

\[ \Pi_{\text{net}} = 10^{3} \times 5.81 \times 3 \times \left[ 3.7 \times 0.5 \times \left( \frac{e.5}{\sin 60} \right)^{2} - \frac{1}{3} \sin 60 \left( \frac{e.5}{\sin 60} \right) \right] \]

\[ \Pi_{\text{net}} = 2,453 \times 10^{3} \, N \cdot m = 2,453 \, kN \cdot m \]

4.4

New water height: the only parameter in eq (1) which changes is \( R_{\text{max}} \)

\[ F_{\text{net} 2} = 2 \, F_{\text{net} 1} \]

\[ \rho g L_{1} \left( Z_{\text{max}} \right) \, R_{\text{max}} - \frac{1}{2} \, \sin \theta \, R_{\text{max}}^{2} \]

\[ = 2 \, \rho g L_{1} \left( Z_{\text{max}} \right) \, R_{\text{max}} - \frac{1}{2} \, \sin \theta \, R_{\text{max}}^{2} \]
\[ Z_{\text{max}2} - \frac{1}{2} \sin \theta \ R_{\text{max}} = 2 \int \left( \frac{Z_{\text{max}1}^2}{2} - \frac{1}{2} \sin \theta \ R_{\text{max}} \right) \]

\[ Z_{\text{max}2} = 2 \cdot Z_{\text{max}1} - \frac{1}{2} \sin \theta \ R_{\text{max}} \]

\[ Z_{\text{max}2} = 2 \times 3.7 - \frac{1}{2} \sin 60 \times \frac{8.5}{\sin 60} \]

\[ Z_{\text{max}2} = 6.15 \text{ m} \]

The corresponding water level increase is 2.45 m.
\( A_1 = 7 \text{ m}^2 \)

\( V_1 = 40 \text{ km/h} = 11.1 \text{ m/s} = V_1 \)

\( T = 300 \text{ } ^\circ \text{C} \)

\( \rho = 1 \text{ bar} \)

\[ \rho = \frac{p}{RT} = \frac{10^5}{8.87 \times (300 + 273.15)} = 0.607 \text{ kg/m}^3 = \rho \]

\( A_2 = 1.5 \text{ m}^2 \)
\[ \mathbf{V}_{1} = \begin{pmatrix} V_{1x} \\ V_{1y} \end{pmatrix} = \begin{pmatrix} 11.7 \times \cos 90° \\ 11.7 \times \sin 90° \end{pmatrix} \]

\[ \mathbf{V}_{1} = \begin{pmatrix} 7.142 \\ 8.512 \end{pmatrix} \text{ m/s} \]

\[ \dot{m} = \rho_{1} V_{1x} A_{1} = \rho_{1} V_{1x} A_{1} \]

\[ = 0.607 \times 7.142 \times 7 \]

\[ \dot{m} = 30.35 \text{ kg/s} \]

**mass conservation:** eq (2) with steady flow \( (d/dt = 0) \)

\[ 0 = \int_{cs} \rho \left( \mathbf{V}_{\text{rel}} \cdot \mathbf{n} \right) dA = \dot{m}_{\text{net}} \]

\[ = \rho_{1} V_{1x} A_{1} - \rho_{2} V_{2x} A_{2} \]

\[ V_{e1} = \frac{\rho_{1} A_{1}}{A_{2}} V_{1x} \]
\[ V_{x,y} = \frac{7}{1.5} \times 7.142 \]

\[ V_{x,y} = 33.33 \text{ m/s} \]

so:

\[ \vec{V}_2 = \begin{pmatrix} 0 \\ 33,33 \end{pmatrix} \text{ m/s} \]

Now, momentum equation (3)

\[
\vec{F}_{net} = \frac{d}{dt} \iint \rho \vec{V} \, dV + \iint \rho \vec{V} \left( \vec{V}_{sea} \cdot \vec{n} \right) \, dA
\]

\[ = \iint \rho \vec{V} \vec{V}_2 \, dA \quad - \iint \rho \vec{V} \vec{V}_1 \, dA \]

uniform inlet + outlet:

\[ = \rho_2 V_2 A_2 \vec{V}_2 - \rho_1 V_1 A_1 \vec{V}_1 \]

\[ \vec{F}_{net} = \dot{m} \left( \vec{V}_2 - \vec{V}_1 \right) \]
\[
F_{set} = \bar{m} \begin{pmatrix}
V_e \gamma - V_{12c} \\
V_e \gamma - U_{1,\gamma}
\end{pmatrix}
= 30,35 \begin{pmatrix}
0 - 7,142 \\
33,33 - 8,512
\end{pmatrix}
\]

\[
\begin{pmatrix}
-216,8 \\
+753,2
\end{pmatrix} \text{ N}
\]

\[
\vec{F}_{set} = \vec{F}_{\text{fluid/exhaust}} = \begin{pmatrix}
+216,8 \\
-753,2
\end{pmatrix} \text{ N}
\]

The fluid exerts the opposite force on the pipe, i.e.
We want \( F_{\text{net}, Y_B} = 0 \), \( F_{\text{net}, Y_A} \).

For this, many possibilities:

\[
F_{\text{net}, Y_B} = \min \left( V_{2,Y_B} - V_{1,Y_B} \right)
\]

not affected by deflector

\[
V_{1,Y_B} = V_1 \sin \alpha
\quad \text{(eq. 42)}
\]

reduce \( V_{2,Y_B} = \frac{P_{18}}{P_{28}} \frac{A_{18}}{A_{28}} \cdot V_{128} \) \quad \text{(eq. 40)}

One simple solution: increase \( P_{28} \) by cooling, keep all other parameters constant.

Then,

\[
F_{\text{net}, Y_B} = 0, \quad F_{\text{net}, Y_A}
\]

\[
\min \left( V_{2,Y_B} - V_{1,Y} \right) = 0, \quad \min \left( V_{2,Y_A} - V_{1,Y} \right)
\]
\[ V_{2yB} = 0.7 \times V_{8yA} + 0.3 \times V_{1y} \]

\[ V_{2yB} = 0.7 \times 33.33 + 0.3 \times 8.512 \]

\[ V_{2yB} = 35.88 \text{ m/s} \]

With eq (××)

\[ V_{8yB} = \frac{p_1}{p_8} \frac{A_1}{A_2} V_{1x} \]

\[ p_8 = p_1 \frac{A_1}{A_2} \frac{V_{1x}}{V_{2yB}} = 0.607 \times \frac{7}{15} \times \frac{7142}{35.88} \]

\[ p_8 = 0.782 \text{ kg/m}^3 \] (88% increase).

Many other solutions exist! Increase \( \alpha \), tilt outlet to angle \( \beta \), increase outlet area, etc...
6.1 Scale model of bird

A - real bird

B - model

\[ L_A = 1.8 \text{ m} \]
\[ m_A = 5 \text{ kg} \]
\[ V_A = 60 \text{ km/h} \]
\[ V_A = 16.6 \text{ m/s} \]
\[ T_A = 35^\circ C \]

\[ P_A = \frac{P_A}{R T_A} = \frac{10^5}{\text{873 x (35 x 95)}} \]
\[ P_A = 1.1676 \text{ kPa/m}^2 \]
\[ \mu_A = 2.1 \times 10^{-5} \text{ Pa.s} \]

\[ L_B = 0.8 \text{ m} \]
\[ T_B = 15^\circ C \]
\[ P_B = 1.1803 \text{ kPa/m}^2 \]
\[ \mu_B = 1.5 \times 10^{-5} \text{ Pa.s} \]
For identical viscous effects, we reproduce Reynolds number:

\[
\left[ \text{Re} \right]_A = \left[ \text{Re} \right]_B
\]

\[
\frac{\rho_A V_A L_A}{\mu_A} = \frac{\rho_B V_B L_B}{\mu_B}
\]

\[
V_B = V_A \frac{L_A}{L_B} \frac{\rho_A}{\rho_B} \frac{\mu_B}{\mu_A}
\]

\[
= \frac{16.6}{0.8} \frac{118}{116.5} \frac{1.2}{2.1}
\]

\[
V_B = 32.78 \text{ m/s} \quad (118 \text{ km/h})
\]

Wind tunnel velocity for identical \( [\text{Re}] \)

6.2

Since main flow parameter is reproduced \( (\left[ \text{Re} \right]) \),
we have identical force coefficients:

\[
C_{F_A} = C_{F_B}
\]
On real bird, \( \text{Lift} = \text{Weight} \)

On model, \( \text{Lift} = F_{LB} \)

\[
C_{LA} = \frac{F_{LA}}{\frac{1}{2} \rho_A S_A V_A^2} = C_{LB} = \frac{F_{LB}}{\frac{1}{2} \rho_B S_B V_B^2}
\]

\[
F_{LB} = F_{LA} \frac{\rho_B}{\rho_A} \frac{S_B}{S_A} \frac{V_B^2}{V_A^2}
\]

\[
= W_{bird} \frac{\rho_B}{\rho_A} \frac{L_B^2}{L_A^2} \frac{V_B^2}{V_A^2}
\]

\[
= 5 \times 9.81 \times \frac{1.203}{1.169} \times \frac{0.8^2}{1.8^2} \times \frac{38.78^2}{16.6^2}
\]

\[
F_{LB} = 38.8 \text{ N} \quad (\sim 4 \text{ kg})
\]
\[ \text{max } [Re] \text{ would be obtained at low temperature:} \]

Case C at \(-5^\circ C\), 65 cm/h

\[ P_c = \frac{10^5}{e^{977 + (173,15-5)}} = \]

\[ P_c = 1.89 \text{ l/s/m}^2 \]

\[ \mu_c = 1.8 \times 10^{-5} \text{ Pa s} \] (from Fig. 1)

We equate \([Re]\) between Case C in cooled tunnel and Case D in normal atmosphere at unknown speed \(V_D\):

\[ [Re]_C = [Re]_D \]

\[ \frac{P_c}{\mu_c} \frac{V_c}{L_c} = \frac{P_D}{\mu_D} \frac{V_D}{L_D} = \frac{P_A}{\mu_A} \frac{V_D}{L_A} \]

\[ V_D = V_C \frac{\mu_A}{\mu_c} \frac{P_c}{P_A} \frac{L_c}{L_A} \]
\[ V_D = \frac{65}{3} \times \frac{2.1}{1.8} \times \frac{19.95}{1.163} \times \frac{0.8}{1.8} \]

\[ V_D = 10.4 \text{ m/s} \quad (35.4 \text{ km/h}) \]

\[ \frac{F_c}{F_D} = \left( \frac{C_{FC}}{C_{FD}} \right)^{\frac{1}{2}} \left( \frac{\rho_c}{\rho_d} \right)^{\frac{1}{2}} \left( \frac{S_c}{S_d} \right) \left( \frac{V_c}{V_D} \right)^2 = \frac{\rho_c L_c^2 V_c^2}{\rho_d L_D^2 V_D^2} \]

Since \[ [\rho_c]_c = [\rho_d]_D \]

\[ \frac{F_c}{F_D} = \frac{1.895 \times 0.12^3 \times 18.06^2}{1.163 \times 1.8^2 \times 3858^2} = 0.6145 = \frac{F_c}{F_D} \]

- Multiply measured forces by 1.512 to scale to reality.