Hello! You are consulting an examination paper from the archives at https://fluidmech.ninja/.

In the summer semester 2020, the general structure of the examination will be largely the same as in this archive. Nevertheless, because the course content progressively changes from year to year, there are a few differences. In former years,

- The course contained a chapter about compressible air flow (involving tables for air properties) that is no longer part of the course now;

- Conversely, several chapters have been added to the examinable content over the years;

- The course contained a duct flow problem involving a ball fountain ("Kugel fountain") that is no longer part of the course now;

- Viscosity values were read in a different diagram, and may not match values read in the 2020 viscosity diagram;

- Many small updates in the notation had not yet been carried out.

To obtain precise information about the summer semester 2020 examination, consult the dedicated appendix in the lecture notes. If you have questions, contact me as detailed in the course syllabus. Thanks, and good luck in your revisions!

Olivier Cleynen

September 2020
Fluid Mechanics examination — February 1st, 2017

Fluid Mechanics for Master Students

Solve problem 1, plus three other problems among problems 2 to 6.

Duration: 2 h – Use of calculator is authorized; documents are not authorized.

Except otherwise indicated, we assume that fluids are Newtonian, and that:

\[ \rho_{\text{water}} = 1\,000\, \text{kg\,m}^{-3}; \rho_{\text{atm.}} = 1\, \text{bar}; \rho_{\text{atm.}} = 1.225\, \text{kg\,m}^{-3}; T_{\text{atm.}} = 11.3\, ^\circ\text{C}; \]
\[ \mu_{\text{atm.}} = 1.5 \times 10^{-5}\, \text{N\,s\,m}^{-2}; g = 9.81\, \text{m\,s}^{-2}. \] Air is modeled as a perfect gas (\( R_{\text{air}} = 287\, \text{J\,K}^{-1}\,\text{kg}^{-1}; \)
\[ \gamma_{\text{air}} = 1.4; c_{\text{p}} = 1\,005\, \text{J\,kg}^{-1}\,\text{K}^{-1}. \]

Reynolds Transport Theorem:

\[
\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt} \int \int \int_{\text{CV}} \rho b \, dV + \int \int_{\text{CS}} \rho b \, (\vec{V}_{\text{rel}} \cdot \vec{n}) \, dA
\]  

(1)

Mass conservation:

\[
\frac{dm_{\text{sys}}}{dt} = 0 = \frac{d}{dt} \int \int \int_{\text{CV}} \rho \, dV + \int \int_{\text{CS}} \rho \, (\vec{V}_{\text{rel}} \cdot \vec{n}) \, dA
\]  

(2)

Change in linear momentum:

\[
\frac{d(m\vec{V}_{\text{sys}})}{dt} = \vec{F}_{\text{net}} = \frac{d}{dt} \int \int \int_{\text{CV}} \rho \vec{V} \, dV + \int \int_{\text{CS}} \rho \vec{V} \, (\vec{V}_{\text{rel}} \cdot \vec{n}) \, dA
\]  

(3)

Change in angular momentum:

\[
\frac{d(\vec{r}_{\text{Xm}} \wedge m\vec{V})_{\text{sys}}}{dt} = \vec{M}_{\text{net},\text{X}} = \frac{d}{dt} \int \int \int_{\text{CV}} \vec{r}_{\text{Xm}} \wedge \rho \vec{V} \, dV + \int \int_{\text{CS}} \vec{r}_{\text{Xm}} \wedge \rho (\vec{V}_{\text{rel}} \cdot \vec{n}) \vec{V} \, dA
\]  

(4)

Continuity equation:

\[
\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \vec{V} = 0
\]  

(5)

Navier-Stokes equation for incompressible flow:

\[
\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \vec{V}_p + \mu \nabla^2 \vec{V}
\]  

(6)

\[
[\text{St}] \frac{\partial \vec{V}^*}{\partial t^*} + [1] \vec{V}^* \cdot \nabla \vec{V}^* = \frac{1}{[\text{Fr}]^2} \vec{g}^* - [\text{Eu}] \vec{V}^* \rho^* + \frac{1}{[\text{Re}]} \nabla^2 \vec{V}^*
\]  

(7)

in which \([\text{St}] \equiv \frac{L}{V}, [\text{Eu}] \equiv \frac{\rho \, c_{\text{p}}}{\mu / V}, [\text{Fr}] \equiv \frac{V}{\sqrt{g} \, r} \quad \text{and} \quad [\text{Re}] \equiv \frac{\rho \, V \, L}{\mu}.
\]
In a highly-viscous (creeping) steady flow, the drag $F_D$ exerted on a spherical body of diameter $D$ at by flow at velocity $V_\infty$ is quantified as:

$$F_{D\text{sphere}} = 3\pi \mu V_\infty D \quad (8)$$

In cylindrical pipe flow, we accept the flow is always laminar for $[\text{Re}]_D \lesssim 2300$, and always turbulent for $[\text{Re}]_D \gtrsim 4000$. The Darcy friction factor $f$ is defined as:

$$f \equiv \frac{|\Delta p|}{\frac{1}{2} \rho V^2_{av.}} \quad (9)$$

A pump or turbine subjected to a pressure difference $\Delta p$ and a volume flow $\dot{V}$ has a power expressed by:

$$\dot{W} = |\Delta p| \dot{V} \quad (10)$$

Figure 1 quantifies the viscosity of various fluids as a function of temperature, and figure 2 (Moody diagram) quantifies losses in cylindrical pipes.

In boundary layer flow, we accept that transition occurs at $[\text{Re}]_x \approx 5 \cdot 10^{5}$. The shear coefficient $c_f$, a function of distance $x$, is defined based on the free-stream flow velocity $U$:

$$c_{f(x)} \equiv \frac{\tau_{\text{wall}}}{\frac{1}{2} \rho U^2} \quad (11)$$

Exact solutions to the laminar boundary layer along a smooth surface yield:

$$\frac{\delta}{x} = \frac{4.91}{\sqrt{[\text{Re}]_x}} \quad \frac{\delta^*}{x} = \frac{1.72}{\sqrt{[\text{Re}]_x}} \quad (12)$$

$$\frac{\theta}{x} = \frac{0.664}{\sqrt{[\text{Re}]_x}} \quad c_{f(x)} = \frac{0.664}{\sqrt{[\text{Re}]_x}} \quad (13)$$

Solutions to the turbulent boundary layer along a smooth surface yield the following time-averaged characteristics:

$$\frac{\delta}{x} \approx \frac{0.16}{[\text{Re}]_x^{0.25}} \quad \frac{\delta^*}{x} \approx \frac{0.02}{[\text{Re}]_x^{0.25}} \quad (14)$$

$$\frac{\theta}{x} \approx \frac{0.016}{[\text{Re}]_x^{0.7}} \quad c_{f(x)} \approx \frac{0.027}{[\text{Re}]_x^{0.7}} \quad (15)$$
The force coefficient $C_F$ and power coefficient $C_p$ are defined as:

$$C_F \equiv \frac{F}{\frac{2}{3} \rho SV^2} \quad C_p \equiv \frac{\dot{W}}{\frac{1}{3} \rho SV^3}$$  \hspace{1cm} (16)$$

The speed of sound $a$ in a perfect gas is a local property expressed as:

$$a = \sqrt{\gamma RT}$$  \hspace{1cm} (17)$$

The total properties (subscript 0) of a perfect gas are expressed as:

$$T_0 \equiv T + \frac{1}{c_p} \frac{1}{2} V^2 \quad \frac{P_0}{\rho_0} = RT_0$$  \hspace{1cm} (18)$$

In isentropic, one-dimensional flow, we accept that the mass flow $\dot{m}$ is quantified as:

$$\dot{m} = \rho VA = A[Ma]\rho_0 \sqrt{\frac{\gamma}{RT_0}} \left[ 1 + \frac{(y-1)[Ma]^2}{2} \right]^{\frac{y-1}{2(y-1)}}$$  \hspace{1cm} (19)$$

This expression reaches a maximum $\dot{m}_{\text{max}}$ when the flow is choked:

$$\dot{m}_{\text{max}} = \left[ \frac{2}{y+1} \right]^{\frac{y+1}{2(y-1)}} A^* \rho_0 \sqrt{\frac{\gamma}{RT_0}}$$  \hspace{1cm} (20)$$

The properties of air flowing through a converging-diverging nozzle are described in figure 3, and figure 4 describes air property changes through a perpendicular shockwave. You may approximate your table readings to those of the line with the nearest value.

Figure 1 – Viscosity of various fluids at a pressure of 1 bar (in practice viscosity is almost independent of pressure).
Figure 2. A Moody diagram, which presents values for the friction factor $f$ measured experimentally as a function of the diameter-based Reynolds number $Re_D$, for different relative roughness values. In this figure, the pipe diameter is noted $d$.

**Moody Diagram**

Friction Factor

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete, coarse</td>
<td>0.25</td>
</tr>
<tr>
<td>Concrete, new smooth</td>
<td>0.025</td>
</tr>
<tr>
<td>Drawn tubing</td>
<td>0.0025</td>
</tr>
<tr>
<td>Glass, Plastic, Perspex</td>
<td>0.0025</td>
</tr>
<tr>
<td>Iron, cast</td>
<td>0.15</td>
</tr>
<tr>
<td>Sewers, old</td>
<td>3.0</td>
</tr>
<tr>
<td>Steel, mortar lined</td>
<td>0.1</td>
</tr>
<tr>
<td>Steel, rusted</td>
<td>0.5</td>
</tr>
<tr>
<td>Steel, structural or forged</td>
<td>0.025</td>
</tr>
<tr>
<td>Water mains, old</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Friction Factor = \( \frac{2d}{\rho V^2 d \Delta P} \)

Relative Pipe Roughness $\varepsilon$

Reynolds Number, $Re = \frac{\rho V d}{\mu}$
One-dimensional isentropic compressible flow functions for an ideal gas with $k = 1.4$

<table>
<thead>
<tr>
<th>$Ma$</th>
<th>$Ma^*$</th>
<th>$A/A^*$</th>
<th>$P/P_0$</th>
<th>$\rho/\rho_0$</th>
<th>$TT_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1094</td>
<td>5.8218</td>
<td>0.9930</td>
<td>0.9950</td>
<td>0.9980</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2182</td>
<td>2.9635</td>
<td>0.9725</td>
<td>0.9803</td>
<td>0.9921</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3257</td>
<td>2.0351</td>
<td>0.9395</td>
<td>0.9564</td>
<td>0.9823</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4313</td>
<td>1.5901</td>
<td>0.8956</td>
<td>0.9243</td>
<td>0.9690</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5345</td>
<td>1.3398</td>
<td>0.8430</td>
<td>0.8852</td>
<td>0.9524</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6348</td>
<td>1.1882</td>
<td>0.7840</td>
<td>0.8405</td>
<td>0.9328</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7318</td>
<td>1.0944</td>
<td>0.7209</td>
<td>0.7916</td>
<td>0.9107</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8251</td>
<td>1.0382</td>
<td>0.6560</td>
<td>0.7400</td>
<td>0.8865</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9146</td>
<td>1.0089</td>
<td>0.5913</td>
<td>0.6870</td>
<td>0.8606</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.5283</td>
<td>0.6339</td>
<td>0.8333</td>
</tr>
<tr>
<td>1.2</td>
<td>1.1583</td>
<td>1.0304</td>
<td>0.4124</td>
<td>0.5311</td>
<td>0.7764</td>
</tr>
<tr>
<td>1.4</td>
<td>1.2999</td>
<td>1.1149</td>
<td>0.3142</td>
<td>0.4374</td>
<td>0.7184</td>
</tr>
<tr>
<td>1.6</td>
<td>1.4254</td>
<td>1.2502</td>
<td>0.2353</td>
<td>0.3557</td>
<td>0.6614</td>
</tr>
<tr>
<td>1.8</td>
<td>1.5360</td>
<td>1.4390</td>
<td>0.1740</td>
<td>0.2868</td>
<td>0.6068</td>
</tr>
<tr>
<td>2.0</td>
<td>1.6330</td>
<td>1.6875</td>
<td>0.1278</td>
<td>0.2300</td>
<td>0.5556</td>
</tr>
<tr>
<td>2.2</td>
<td>1.7179</td>
<td>2.0050</td>
<td>0.0935</td>
<td>0.1841</td>
<td>0.5081</td>
</tr>
<tr>
<td>2.4</td>
<td>1.7922</td>
<td>2.4031</td>
<td>0.0684</td>
<td>0.1472</td>
<td>0.4647</td>
</tr>
<tr>
<td>2.6</td>
<td>1.8571</td>
<td>2.8960</td>
<td>0.0501</td>
<td>0.1179</td>
<td>0.4252</td>
</tr>
<tr>
<td>2.8</td>
<td>1.9140</td>
<td>3.5001</td>
<td>0.0368</td>
<td>0.0946</td>
<td>0.3894</td>
</tr>
<tr>
<td>3.0</td>
<td>1.9640</td>
<td>4.2346</td>
<td>0.0272</td>
<td>0.0760</td>
<td>0.3571</td>
</tr>
<tr>
<td>5.0</td>
<td>2.2361</td>
<td>25.000</td>
<td>0.0019</td>
<td>0.0113</td>
<td>0.1667</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$2.2495$</td>
<td>$\infty$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3 – Properties of air (modeled as a perfect gas) as it expands through a converging-diverging nozzle. In this figure, the Mach number is noted “$Ma$”, and the perfect gas parameter $\gamma$ is noted $k$. Data also includes the parameter $Ma^* \equiv V/a^*$ (speed non-dimensionalized relative to the speed of sound at the throat).

Figure © Çengel & Cimbala 2010, Fluid Mechanics, 2nd ed., pub. McGraw-Hill
Figure 4 – Properties of air (modeled as a perfect gas) as it passes through a perpendicular shockwave. In this figure, the Mach number is noted “Ma”, and the perfect gas parameter $\gamma$ is noted $k$.

Figure © Çengel & Cimbala 2010, Fluid Mechanics, 2nd ed., pub. McGraw-Hill
Solve problem 1, 
and three other problems among problems 2 to 6.

The following marking guidelines will be used:

- Answers to questions starting with “show that” should be fully-developed and continuous;
- In all other questions, the correct result with the correct unit is enough to obtain full points;
- Illegible or ambiguous answers are always discarded.
1 Governing equation

1.1. Write out the Navier-Stokes equation for incompressible flow (eq. 6) in its fully-developed form in three Cartesian coordinates.

1.2. In which flow conditions can equation (5) for continuity, \( \frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \vec{V} = 0 \), apply?

2 Force due to pressure on an aquarium window

A large aquarium is in use in a museum. It features an inclined wall with a large window, as shown in fig. 5.

The bottom edge of the window stands 2 m below the water surface and 0.4 m above the ground. The window has width \( L_1 = 3 \text{ m} \) and is inclined at an angle \( \theta = 70^\circ \) relative to horizontal. The stationary water exerts a force \( \vec{F}_{\text{pressure}} \) on the window.

2.1. Represent graphically the distribution on each side of the window of the pressure exerted by the water and atmosphere.

2.2. What is the magnitude of the net force \( \vec{F}_{\text{pressure}} \) exerted by the water on the window?

2.3. At which height does the net force \( \vec{F}_{\text{pressure}} \) apply?

2.4. How does the force on the door vary when the atmospheric pressure increases? (briefly justify your answer, e.g. in 30 words or less)
3 Viscometer

An instrument designed to measure the viscosity of fluids is made of two coaxial cylinders (fig. 6). The inner cylinder is immersed in a liquid, and it rotates within the stationary outer cylinder.

![Figure 6 – Sketch of a cylinder viscometer. The width of the gap has been greatly exaggerated for clarity.](Figure CC-0 o.c.)

The height of the liquid container is $H = 80$ cm. The inner cylinder diameter is $D_1 = 20$ cm and the spacing is $\Delta x = 2$ mm.

When the inner cylinder is rotated at $\omega = 150$ rpm, a friction-generated moment $M_{\text{friction}} = 0,9$ N m is measured.

3.1. In the case where the flow in between the cylinders corresponds to the simplest possible flow case (steady, uniform, fully-laminar), show that the shear $\tau_1$ exerted on the inner cylinder surface is:

$$\tau_1 = \frac{\mu \omega D_1}{2 \Delta x} \quad (21)$$

3.2. What is the viscosity of the fluid?

3.3. Would the measured moment be larger if the fluid was replaced with a non-Newtonian fluid? (briefly justify your answer, e.g. in 30 words or less)
4 Wind tunnel model of a racing motorcycle

A team races a motorcycle in a competition. The motorcycle is 1.9 m long, has a 105 kW engine, and a mass of 150 kg. At full power, the motorcycle and its driver can reach a top speed of 260 km h\(^{-1}\).

In order to test different aerodynamic configurations for the motorcycle, the team wishes to print a model for use in a wind tunnel. The team is evaluating two options: using a 50% model or a (smaller) 25% model.

4.1. What would be the frontal area of each model in relation to the frontal area of the real motorcycle?

4.2. What would be the volume of each model, compared to the volume of the real motorcycle?

4.3. How much less weight would the 25% model have than the 50% model?

In the end, the team decides to use a 50% model, as represented in fig. 7.

4.4. If the ambient atmospheric conditions cannot be changed, which flow speed in the wind tunnel is required, so that the air flow around the real motorcycle at maximum speed is reproduced around the model?

4.5. What would then be the power dissipated as friction by the model?

In practice, the racing regulations limit the maximum air speed that can be used during wind tunnel tests. The maximum authorized wind tunnel velocity is 55 m s\(^{-1}\).

Figure 7 – A 50% model of a racing motorcycle is photographed in the test section of a wind tunnel.

Photo CC-by-sa by Patrick Haas, CMEF Geneva
4.6. The team considers modifying the air temperature to compensate for the limit in the air speed. If the temperature in the tunnel can be controlled between $-10^\circ$C and $45^\circ$C, and the pressure can be reduced to 80% of atmospheric pressure, what is the maximum race-track speed of the original motorcycle that can be reproduced in the wind tunnel?

4.7. In that case, what is the ratio between drag force measurements on the model and the corresponding drag force on the real motorcycle?

5 Hydraulic power required for pumping

A cylindrical pipe connects two reservoirs, as shown in fig. 8. A pump is used to transfer water from one reservoir to the other.

The pipe has diameter $D = 0.25$ m and wall roughness $\epsilon = 0.25$ mm; the water temperature is $20^\circ$C. It is required that a volume flow rate of $12$ L s$^{-1}$ is generated through the pipe.

5.1. Represent qualitatively (i.e. without numerical data) the water pressure distribution along the pipe length.

5.2. What is the hydraulic power required to pump the water through the pipe?

5.3. What would be the change in power if the pipe diameter was halved, but the volume flow was kept identical?

Figure 8 – A pipe linking two reservoirs, with water flowing from left to right. A pump is positioned at the inlet of the pipe. In this figure, vertical distances and the pipe diameters are greatly exaggerated for clarity.
6 Exhaust duct

The exhaust of a ship engine exits the engine room through a vertical pipe; at the end of the pipe, the exhaust gases are deflected into the atmosphere with a deflector represented in fig. 9.

![Figure 9 - Deflector used to reject exhaust gases from an engine into the atmosphere. Gases enter vertically at velocity } V_1 \text{ and exit at an angle } \theta \text{ with velocity } V_2.

The deflector is fed with a vertical jet of hot gases with a quasi-uniform velocity profile; the speed is } V_1 = 80 \text{ km h}^{-1}, \text{ temperature } T_1 = 450 \text{ } ^\circ \text{C} \text{ and the pressure is atmospheric. As the exhaust gases travel through the pipe, their heat losses are negligible. Gases are rejected with an angle } \theta = 30^\circ \text{ angle relative to the vertical. The inlet diameter is } D_1 = 1.8 \text{ m and the outlet sectional area, measured in the vertical plane, is } A_{\text{outlet}} = 15 \text{ m}^2.

6.1. What is the force exerted on the pipe by the exhaust gases?

6.2. Describe and quantify a modification to the deflector that would reduce the vertical component of the force by 20%.

6.3. Would the force be modified if the heat losses of the gases became significant, so that the exhaust temperature } T_2 \text{ became significantly less than their inlet temperature } T_1? \text{ (briefly justify your answer, e.g. in 30 words or less)