Hello! You are consulting an examination paper from the archives at https://fluidmech.ninja/.

In the summer semester 2020, the general structure of the examination will be largely the same as in this archive. Nevertheless, because the course content progressively changes from year to year, there are a few differences. In former years,

- The course contained a chapter about compressible air flow (involving tables for air properties) that is no longer part of the course now;
- Conversely, several chapters have been added to the examinable content over the years;
- The course contained a duct flow problem involving a ball fountain (“Kugel fountain”) that is no longer part of the course now;
- Viscosity values were read in a different diagram, and may not match values read in the 2020 viscosity diagram;
- Many small updates in the notation had not yet been carried out.

To obtain precise information about the summer semester 2020 examination, consult the dedicated appendix in the lecture notes. If you have questions, contact me as detailed in the course syllabus. Thanks, and good luck in your revisions!

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September 2020
1) Governing equation

\[
\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = \rho g_x - \frac{\partial p}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]
\]

\[
\rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] = \rho g_y - \frac{\partial p}{\partial y} + \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right]
\]

\[
\rho \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] = \rho g_z - \frac{\partial p}{\partial z} + \mu \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right]
\]

2) It applies in all incompressible flows of Newtonian fluids.

There are no other restrictions.
2. Design of a supersonic wind tunnel

\[ A_T = 0.08^2 = 6.4 \times 10^{-3} \text{ m}^2 = A_T \]

\[ A_{out} = 0.13^2 = 1.2765 \times 10^{-2} \text{ m}^2 = A_{out} \]

\[ A_L = 9.8 \times 10^{-3} \text{ m}^2 \]

2.1

Flow subsonic everywhere, highest \( p_o \) : flow is choked and we have \( \frac{[\rho_a]}{[\rho_a]} = 1 \)

in this case, \( A_T = A^* \)

so \( \frac{A_{out}}{A_T} = \frac{A_{out}}{A^*} = \left( \frac{0.85}{0.08} \right)^2 = 0.0952 \approx 8 \)

From fig. 3 we read \( [\rho_a]_{nf} \approx 0.3 \) and \( \frac{P_{out}}{P_o} = 0.9335 = \frac{P_b}{P_o} \)

\[ P_o = \frac{1}{0.9335} = 1.064 \text{ bar} = \frac{P_b}{P_o} \]
Flow is isentropic until \( \text{PR} \) \( (\text{upstream of shock}) \)

\[
\frac{A_{u}}{A_{s}} = \frac{52 \times 10^{-3}}{64 \times 10^{-3}} = 1.4375 \approx 1.439
\]

From fig. 3, we read \( [\alpha]_{f} = 1.8 \) \( \frac{\pi}{\alpha} \)

From fig. 4, we read the pressure jump from \( \text{PR} \) to \( \text{PD} \) \( (\text{downstream of shock}) \)
8.2 Shock-less expansion to supersonic flow:
we still have \( \frac{A_{at}}{A} = 2 \) but look for
the supersonic flow case: \([N_a]_{at} = 2,2\)

we read \( \frac{P_{at}}{p_0} = 0,0335 = \frac{P_b}{p_0} \)

\[ p_0 = \frac{1}{0,0335} = 10,7 \text{ bar} = p_0 \]

8.3 We have \([N_a]_{at} = 2,2 = \frac{V_{at}}{a_{at}} = \frac{V_{at}}{\sqrt{\gamma R T_{at}}} \)

\[ V_{at} = \frac{[S_{at}]}{a_{at} \left[ \gamma R T_{at} \right]^\frac{1}{2}} \]

\[ = 2,2 \left[ \frac{14 \times 287 \times (2 + \gamma \times 333)}{0,5} \right] \]

\[ V_{at} = 733,4 \text{ m/s} \quad (2662 \text{ km/h}) \]

8.4 \( \dot{m} = \rho_{at} \frac{V_{at}}{A_{at}} \)

\[ = \frac{P_{at}}{R T_{at}} \frac{V_{at}}{A_{at}} = \]

\[ = \frac{10^5}{887 \times 333,4} \times 733,4 \times 1,2769 \times 10^{-2} \]

\[ \dot{m} = 11,7 \text{ kg/s} \]
\[ \frac{P_{id}}{P_{iu}} = 3.6133 \]

\[ P_{id} = 3.6133 \cdot P_{iu} \]

pressure increase across shock

\[ T_{id} = 1.5316 \cdot T_{iu} \]

2.8

The reservoir temperature is the total temperature \( T_0 \). Decreasing it reduces the amount of energy available for the air to accelerate. The air will therefore shock earlier in the tunnel; it may even remain subsonic all along, if \( \frac{T_0}{T} \) is low enough to become smaller than \( \frac{T_0}{T^*} \).
### Wind tunnel model of motorcycle

<table>
<thead>
<tr>
<th>Motorcycle (1)</th>
<th>Model (2)</th>
<th>Model (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1 = 1.8 \text{ m}$</td>
<td>$\frac{L_2}{L_1} = 0.75$</td>
<td>$\frac{L_3}{L_1} = 0.5$</td>
</tr>
<tr>
<td>$V_{\text{top1}} = 880 \text{ km/h}$</td>
<td>$= 27.8 \text{ m/s}$</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.1

\[
\frac{S_{f2}}{S_{f1}} = \frac{L_2^2}{L_1^2} = 0.75^2 = 0.5625 = \frac{S_{f2}}{S_{f1}}
\]

(Total surface of first model is 56% of full-size motorcycle)

\[
\frac{S_{f3}}{S_{f1}} = 0.5^2 = 0.25 = \frac{S_{f2}}{S_{f1}}
\]
3.2 \[ \frac{r_2}{r_1} = \left( \frac{L_2}{L_1} \right)^2 = 0.75^3 = 0.4218 = \frac{r_2}{r_1} \]

(volume is 42.18\% of original)

\[ \frac{r_3}{r_1} = 0.185 \]

2.3

Models printed with same material:

\[ \frac{m_3}{m_2} = \frac{r_3^3}{r_2^3} = \frac{0.75^3}{0.75^3} = 0.8563 = \frac{m_3}{m_2} \]

model 3 requires 30\% of the material of model 2 (3.4 times less weight and material)

3.4

To scale the flow, we wish to have identical Reynolds number:

\[ [Re]_3 = [Re]_1 \]
\[
\frac{p_3}{\mu_3} \frac{V_3}{L_3} = \frac{p_1}{\mu_1} \frac{V_1}{L_1}
\]

Some atmospheric conditions: \( p_1 = p_3 \)
\( \mu_1 = \mu_3 \)

If \( V_1 = V_{1,op} \)
\[
V_{3,op} \frac{L_3}{L_3} = V_{1,op} \frac{L_1}{L_3} = 77.8 \times \frac{1}{0.5}
\]
\[
V_{3,op} = 155.6 \text{ m/s} = (560 \text{ km/h})
\]

This flow is adequately scaled, flow coefficients are identical:

\( C_{p3} = C_{p4} \)
\[
\frac{W_{\text{max 3}}}{\frac{1}{2} f_3 \frac{S_3}{4} V_3^3} = \frac{W_{\text{max 1}}}{\frac{1}{2} f_1 \frac{S_1}{4} V_1^3}
\]

with identical atmospheric conditions, \( f_1 = f_2 \)

\[
W_{\text{max 3}} = \frac{S_3 \cdot V_3^3}{S_1 \cdot V_1^3} \cdot W_{\text{max 1}}
\]

\[
= \left( \frac{L_3}{L_1} \right)^2 \left( \frac{L_1}{L_3} \right)^3 \hat{W}_{\text{max 1}}
\]

\[
= \frac{L_1}{L_3} \hat{W}_{\text{max 1}}
\]

\[
W_{\text{max 3}} = 2 \hat{W}_{\text{max 1}} = 2 \times 110 \text{ kW}
\]

\[
\hat{W}_{\text{max 3}} = 220 \text{ kW (\sim 300 hp)}
\]

3.6 \[ \text{now } V_{3,\text{max}} = 65 \text{ m/s} \]

To maximize \((Re)_3^2\) (in &minel), we want low \(f_3\),

Therefore \(f_3 \sim \text{flow} \)
\[ p_3 = \frac{p_3}{RT_3} = \frac{10^5}{86.7 \times 268.15} = 1,283.4 \text{ kPa/m}^3 = \rho_3 \]

\[ p_1 = 1,825 \text{ kPa/m}^2 \quad \text{(standard)} \]

Now, maintaining physical similarity with Reynolds number:

\[ \frac{Re_3}{Re_1} = \left( \frac{Re_1}{Re_3} \right) \]

\[ \frac{\frac{V_3}{\mu_3} L_3}{\mu_3} = \frac{p_1 V_{1\text{max}} L_1}{\mu_3} \]

\[ l_{1\text{max}} = \frac{L_3}{L_1} \cdot \frac{p_3 \text{max}}{p_1} \cdot V_{3\text{max}} \quad \text{(assuming constant viscosity)} \]

\[ = 0.5 \times \frac{1,283.4}{1,825} \times 65 \]

\[ V_{1\text{max}} = 34.5 \text{ m/s} \quad (124 \text{ km/h}) \]
3.7 If dynamic similarity is maintained, force coefficients are the same:

\[ C_{F_3} = C_{F_1} \]

\[ \frac{F_3}{\frac{1}{2} \rho_3 S_{f_3} V_3^2} = \frac{F_1}{\frac{1}{2} \rho_1 S_{f_1} V_1^2} \]

\[ \frac{F_2}{F_1} = \frac{S_{f_2}}{S_{f_1}} \frac{\rho_3}{\rho_1} \frac{V_3^2}{V_1^2} = \frac{L_3^2}{L_1^2} \frac{\rho_3}{\rho_1} \frac{L_1^2}{L_3^2} \frac{\rho_1}{\rho_3} \]

\[ = \left( \frac{L_3}{L_1} \right)^2 \frac{\rho_3}{\rho_1} \frac{L_1}{L_3} \left( \frac{\rho_1}{\rho_3} \right) \]

\[ = \frac{\rho_1}{\rho_3} = \frac{1.225}{1.2894} \]

\[ \frac{F_2}{F_1} = 0.942 \]
4. Piping for hydraulic turbine

\[ D_1 = 2.2 \text{ m} \quad | \quad \Delta z_1 = -4 \text{ m} \]
\[ \varepsilon_1 = 0.03 \text{ m/m} \quad | \quad L_1 = 100 \text{ m} \]

\[ K_L = 0.7 \]

\[ D_2 = 1.1 \text{ m} \quad | \quad \Delta z_2 = -4 \text{ m} \]
\[ \varepsilon_2 = 0.02 \text{ m/m} \quad | \quad L_2 = 100 \text{ m} \]

\[ v^2 = 8.5 \text{ m/s} \]

\[ \mu_{\text{eff}} = 1 \times 10^{-3} \text{ Pa.s} \]

4.1 First half of pipe

\[ V_{\text{avg}} = \frac{v^2}{S_1} = \frac{v^2 \rho}{\pi K_L^2} = \frac{2v^3}{\pi \times \left(\frac{v^2}{2}\right)} \]

\[ V_{\text{avg}} = 0.605 \text{ m/s} \]
\[ [Re]_{D_1} = \frac{f_1 V_{1C} D_1}{\nu} = \frac{10^3 \times 0.605 \times 2.2}{10^{-3}} \]

\[ [Re]_{D_1} = 1,231 \times 10^6 \]

\[ \frac{e_1}{D_1} = \frac{0.03 \times 10^{-2}}{\nu} = \frac{0.03 \times 10^{-3}}{D_1} \]

From Moody diagram:

\[ f_1 = 0.0115 \]

\[ \left. \Delta p \right|_{\text{fluid 1}} = f_1 \left( \frac{1}{2} \rho V_{1C}^2 \frac{L_1}{D_1} \right) \]

\[ = 0.0115 \times 0.15 \times 10^3 \times 0.605^2 \times \frac{100}{2.2} \]

\[ \left. \Delta p \right|_{\text{fluid 1}} = 85.67 \text{ Pa} \]

\[ \Delta p_{\text{air 1}} = \rho g D_2 l = 10^3 \times 9.81 \times (4.4) \]

\[ \Delta p_{\text{air 1}} = +33.2 \text{ kPa} \]
in contraction:

\[ \Delta p|_{\text{loss}} = \frac{1}{2} \rho V_{100}^2 k_L \]

\[ = 0.5 \times 10^2 \times 0.605^2 \times 0.5 \]

\[ \Delta p|_{\text{loss}} = 164.7 \text{ Pa} \]

\[ \Delta p|_{\text{vel.}} = -\frac{1}{2} \rho \left( V_{200}^2 - V_{100}^2 \right) \]

\[ V_{200} = \frac{3}{2} \frac{E}{\rho} = \frac{3 \times 2}{2 \times 0.5} \]

\[ V_{200} = 2.42 \text{ m/s} \]

\[ \Delta p|_{\text{vel.}} = -0.5 \times 10^2 \times \left( 2.42^2 - 0.605^2 \right) \]

\[ \Delta p|_{\text{vel.}} = -2,246 \text{ kPa} \]
Second half of pipe:

\[
\left[ \frac{Re}{D_2} \right]_{O_2} = \frac{P_2 \frac{V_{2av}}{D_2}}{\mu_2} = \frac{10^3 \times 2.42 \times 1.1}{10^{-3}}
\]

\[
\left[ Re \right]_{O_2} = 2.662 \times 10^6
\]

\[
\frac{\varepsilon_2}{D_2} = \frac{0.03 \times 10^{-3}}{1.1} = 2.73 \times 10^{-5} = \frac{\varepsilon_2}{O_2}
\]

From Moody diagram: \( f_2 = 0.011 \)

\[
\frac{\Delta P}{(\text{static})_{O_2}} = f_2 \frac{1}{2} \frac{P_2}{V_{2av}} \frac{V^2}{L_2} \frac{D_2}{O_2}
\]

\[
= 0.011 \times 500 \times 2.42^2 \times \frac{100}{11}
\]

\[
\frac{\Delta P}{(\text{static})_{O_2}} = 2.988 \text{ kPa}
\]

\[
\Delta P_{O_2} = \Delta P_{static} = +352 \text{ kPa}
\]
\[
\begin{align*}
\left[ \text{Re} \right]_{D1} &= \frac{\rho_1 V_{10} D_1}{\mu_1} = \frac{10^3 \times 6.065 \times \varepsilon_2}{10^{-3}} \\
\left[ \text{Re} \right]_{D1} &= 1.331 \times 10^6
\end{align*}
\]

\[
\frac{e_1}{D_1} = \frac{0.03 \times 10^{-2}}{8.72} = 1.264 \times 10^{-5} = \frac{\varepsilon_1}{D_1}
\]

from Moody diagram:

\[
\tau = 0.0115
\]

\[
\left. \Delta p \right|_{\text{Hadya 1}} = \frac{1}{2} \rho V^2_{10} \frac{L_1}{D_1}
\]

\[
= 0.0115 \times 0.15 \times 10^3 \times 0.605^2 \times \frac{100}{2.2} = 85.67 \text{ Pa}
\]

\[
\Delta p_{\text{eff 1}} = \rho g D_2 = 10^3 \times 9.81 \times 4.4
\]

\[
\Delta p_{\text{eff 1}} = +33.2 \text{ kPa}
\]
Summary of gauge pressures (pressure minus atm. pressure)

\[ p_A = \rho \cdot g \cdot z_A = 10^3 \times 9.81 \times 5 \quad (\text{gauge}) \]

\[ p_A = 8,795 \text{ kPa} \quad (\text{neglecting drop due to velocity}) \]

\[ p_B = p_A + \Delta p_{\text{diff}} - \left| \Delta p \right|_{\text{friction}} \]

\[ = 8,898 + 29.2 - 85.67 \times 10^{-3} \]

\[ p_B = 8,923 \text{ kPa} \]

\[ p_C = p_B - \left| \Delta p \right|_{\text{loss}} + \Delta p_{\text{velocity}} \]

\[ = 127,43 \times 10^3 - 164.7 + (-274.6 \times 10^3) \]

\[ p_C = 124,78 \text{ kPa} \]

\[ p_D = p_C - \left| \Delta p \right|_{\text{friction}2} + \Delta p_{\text{diff2}} \]

\[ = 124,78 \times 10^3 - 21,378 \times 10^3 + 161.67 \times 10^3 \]

\[ p_D = 161.67 \text{ kPa} \]
\[ \Delta P_{\text{turbine}} = P_g \Delta z_{\text{turbine}} + |\Delta P|_{\text{t}} + |\Delta P|_{\text{loss}} + |\Delta P|_{\text{z}} \]

\[ = 10^3 \times 9,81 \left[ 11 - (9 + 8) \right] + 95,67 + 164,7 + 2,98 \times 10^3 \]

\[ \Delta P_{\text{hub}} = -55,691 \text{ kPa} \]
\[ \dot{W} = \left| \Delta p \right| \dot{V} \]

\[ = 55,671 \times 10^3 \times 2.2 \]

\[ \dot{W}_{hyd} = 128 \text{ kW} \]

4.4 The power would not change, since neither the value of \( \rho_1 \), \( \rho_2 \), nor the pressure passes would change.
5. Drag on a flat surface

\[ u_1 = U = 95 \text{ m/s} \]

\[ u_e = U \left( \frac{Y}{E} \right)^{\frac{1}{6}} \quad \text{with} \quad E = 2 \text{ cm} \]

\[ L_1 = 0.5 \text{ m} \]

\[ L_2 = 0.8 \text{ m} \]

---

The phenomenon is symmetrical; we represent only one half of the control volume here.

From eq. 8 we want to find an expression for \( h \) :

\[
 D = \frac{d}{d \xi} \int_C \rho \, dF + \int_C \rho \left( \mathbf{V}_t \cdot n \right) \, dA + \int_C \rho \left( \mathbf{V}_t \cdot t \right) \, dA
\]

\text{steady flow}
\[ O = \int \int \rho v_1 \, dA - \int \int \rho v_2 \, dA \]

\[ = -\int \int \rho v_1 \, L_2 \, dy - \int \int \rho v_2 \, L_2 \, dy \]

\[ O = \rho L_2 \left( \int_0^{h_1} u_1 \, dy - \int_0^{h_2} u_2 \, dy \right) \]

since \( u_1 \) independent from \( \gamma \) \((\sigma = U)\):

\[ O = u_1 \, h_1 - \int_0^{h_2} u_2 \, dy \]

\[ h_1 = \frac{1}{U} \int_0^{h_2} u_2 \, dy \]

Now, from eq. 3

\[ \bar{F}_{nat} = \frac{d}{dt} \int \int \rho V \, dV + \int \int \rho (\bar{V}_{int} \cdot \bar{u}) \bar{V} \, dA \]

\[ \text{C.V.} \]

\[ \text{C.S.} \]

\[ \text{(skecdy F.C.)} \]
Taking only the \( \alpha \) component,

\[
F_{\alpha \alpha} = - \int_{\text{in}} \rho \, u_1 \, V_{1 \alpha} \, dA + \int_{\text{out}} \rho \, u_2 \, V_{2 \alpha} \, dA
\]

since \( V_{1 \alpha} = u_1 \), and \( V_{2 \alpha} = u_2 \)

\[
F_{\alpha \beta} = - \int_{\text{in}} \rho \, u_1 \, u_2 \, l_2 \, dy + \int_{\text{out}} \rho \, u_2 \, u_2 \, l_2 \, dy
\]

\[
= \rho l_2 \, x \left[ \int_{0}^{1} u_1^2 \, dy + \int_{0}^{\delta} u_2^2 \, dy \right]
\]

\[
\sigma_{\alpha \alpha} = 2 \rho l_2 \left[ - U^2 \, u_1 + \int_{0}^{\delta} u_2^2 \, dy \right]
\]

\[
\text{insert eq. } (6)
\]

\[
F_{\alpha \beta} = 2 \rho l_2 \left[ - U^2 \, u_1 + \int_{0}^{\delta} u_2^2 \, dy \right]
\]

\[
F_{\alpha \beta_{\delta}} = \left[ \left( \delta \right) \left( u_2^2 - U u_2 \right) \right] dy
\]
In eq. 21, we have:

\[ u_2 = U \delta^{-1/6} \gamma^{1/2} \]

Then:

\[ F_{\text{rel}} = 2 \rho L_2 \int_0^\delta \left( U^2 \delta^{-1/2} \gamma^{1/3} - U^2 \delta^{-1/2} \gamma^{1/6} \right) dy \]

\[ = 2 \rho L_2 U^2 \left[ \frac{\delta^{-1/3}}{1 + \frac{1}{3}} \gamma^{1/3} - \frac{\delta^{-1/6}}{1 + \frac{1}{6}} \gamma^{1/6} \right]_0^\delta \]

\[ = 2 \rho L_2 U^2 \left[ \frac{3}{4} \delta^{-1/3} \delta^{1/3} - \frac{6}{7} \delta^{-1/6} \delta^{1/6} \right] \]

\[ = 2 \rho L_2 U^2 \left[ \frac{3}{4} - \frac{6}{7} \right] \delta = 2 \rho L_2 U^2 \left( \frac{-3}{28} \right) \delta \]

\[ F_{\text{rel}} = -\frac{3}{14} \rho L_2 U^2 \delta \]

5.2

\[ F_{\text{rel}} = -\frac{3}{14} \times 1925 \times 0.6 \times 25^2 \times 8 \times 10^{-2} \]

\[ F_{\text{rel}} = -2162.5 \ N \]
So, the drag force on the plate is

\[
\mathbf{F}_{\text{Drag}} = - \mathbf{F}_{\text{Rel}} = +2,625 \, \text{N}
\]  
(positive x-direction)

\[\mathbf{W} = - \mathbf{F}_{\text{Rel}} \cdot \mathbf{V} = +2,625 \times 85\]

\[\mathbf{W} = 65,635 \, \text{Watt}\]

5.3

This power is transferred back to the air as heat, causing an (imperceptibly small) temperature increase.

5.4

While \( L_1 \) does not appear in eq. 81 directly, it influences the value of \( \delta \). The larger \( L_1 \), the thicker the boundary layer. With \( \delta \) increasing, \( F_{\text{rel}} \) and thus the drag will increase.

5.5
Forces on a water tank panel

\[ z = h_2 - r \sin \theta \]

\[ = (h_1 + l_2 \sin \theta) - r \sin \theta \]
\[ F_{\text{net}} = \int \int p_{\text{net}} \, dA \]

\[ = \int_{0}^{L_2} l_2 \, p_{\text{net}} \, L_3 \, dr \]

\[ = \int_{0}^{L_2} p g z \, L_3 \, dr \]

\[ = \int_{0}^{L_2} p g \left[ (H_1 + l_2 \sin \theta) - r \sin \theta \right] \, L_3 \, dr \]

\[ = p g \, L_3 \, \left[ \int_{0}^{L_2} \left( H_1 + l_2 \sin \theta \right) - r \sin \theta \right] \, dr \]

\[ = p g \, L_3 \, \left[ \int_{0}^{L_2} \left( H_1 + l_2 \sin \theta \right) - \frac{1}{2} \sin \theta \, r^2 \right] \, dr \]

\[ F_{\text{net}} = p g \, L_3 \, \left[ \left( H_1 + l_2 \sin \theta \right) L_2 - \frac{1}{2} \sin \theta \, L_2^2 \right] \]
\[ F_{\text{net}} = 10^3 \times 3.81 \times 0.6 \left[ (1 + 2 \sin 35) \times 2 - 0.5 \times \sin 35 \times 2^2 \right] \]

\[ = 18,584 \times 10^3 \text{ N} \]

\[ \boxed{F_{\text{net}} = 18,584 \text{ kN}} \]

(\sim 18,584 \text{ kN})

\[ \text{6.2} \]

\[ \Phi_{\text{net}} = \int r f_{\text{net}} \, dA \quad \text{(net moment around bottom edge of panel)} \]

\[ = \int_{0}^{L_2} \left[ (h_1 + L_2 \sin \theta) r - r^2 \sin \theta \right] \, dr \]

\[ = p g L_3 \left[ \frac{1}{2} \left( h_1 + L_2 \sin \theta \right) L_2^2 - \frac{1}{3} \sin \theta \, L_2^3 \right] \]

\[ \boxed{\Phi_{\text{net}} = p g L_3 \left[ \frac{1}{2} \left( h_1 + L_2 \sin \theta \right) L_2^2 - \frac{1}{3} \sin \theta \, L_2^3 \right]} \]
\[ \tau_{\text{net}} = 10^3 \times 3.81 \times 0.6 \left[ \frac{1}{2} \left( 1 + 2 \sin 35^\circ \right) \times 2^2 \right] \]

\[ = \frac{16,873 \times 10^3}{N_m} \]

\[ \Omega_{\text{net}} = 16,873 \text{ kN.m} \]

Now, \[ \frac{\tau_{\text{net}}}{F_{\text{net}}} = R \]

\[ R = \frac{16,873 \times 10^3}{18,584 \times 10^3} = 0.875 \text{ m} = R \]
if \( H = -k_1 t \)

then \( L_2 = -k_2 t \) and \( H_1 = -k_3 t \)

and \( H_2 = H_1 + L_2 \sin \theta \) decreases with time

so \( F_\text{tot} \) will decrease approximately as so: