

Fluid Dynamics

Chapter 10 – Flow near walls

last edited September 19, 2020
by Olivier Cleynen – <https://fluidmech.ninja/>

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These notes are based on textbooks by White [22], Çengel & al.[25], Munson & al.[29], and de Nevers [17].

10.1 Motivation

In this chapter, we focus on fluid flow close to solid walls. In these regions, viscous effects dominate the dynamics of fluids. This study should allow us to answer two questions:

- How can we quantify shear-induced friction on solid walls?
- How can we describe and predict flow separation?

10.2 The concept of boundary layer

10.2.1 Rationale

At the very beginning of the 20th century, Ludwig Prandtl observed that for most ordinary fluid flows, viscous effects played almost no role outside of a very small layer of fluid along solid surfaces. In this area, shear between the zero-velocity solid wall and the outer flow dominates the flow structure. He named this zone the *boundary layer*.^w

We indeed observe that around any solid object within a fluid flow, there exists a thin zone which is significantly slowed down because of the object's presence. This deceleration can be visualized by measuring the velocity profile (fig. 10.1).

The boundary layer is a *concept*, a thin invisible layer whose upper limit (termed δ , as we will see in §10.2.3) is defined as the distance where the fluid velocity in direction parallel to the wall is 99 % of the outer (undisturbed) flow velocity.

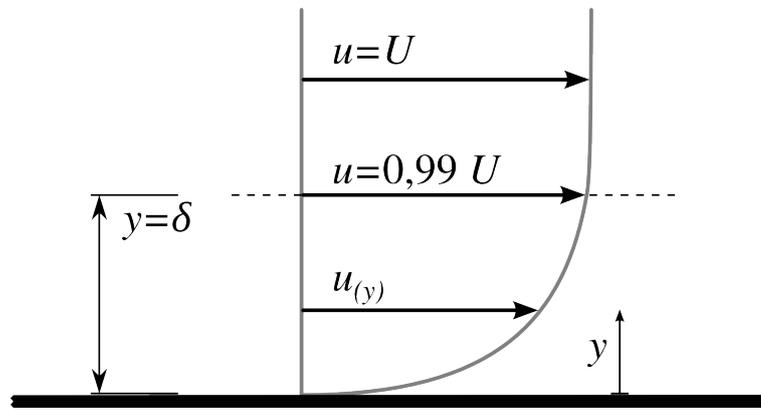


Figure 10.1: A typical velocity profile in a boundary layer. Only the horizontal component of velocity ($V_x = u$) is represented.

Figure CC-BY Olivier Cleynen

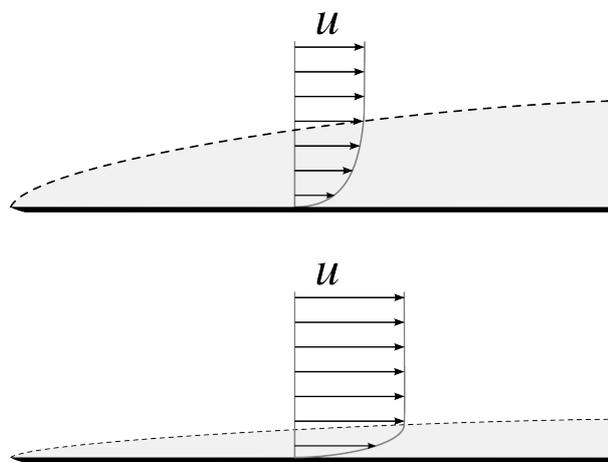


Figure 10.2: The thickness of the boundary layer depends strongly on the faraway incoming flow velocity U_∞ .

Figure CC-BY-SA Olivier Cleynen & Commons User:Flanker

Upon investigation, we observe that the boundary layer thickness depends strongly on the main flow characteristics. In particular, it *decreases* when speed increases or when viscosity is decreased (fig. 10.2).

As we travel downstream along a boundary layer, we observe experimentally that the flow regime is always laminar at first. Then, at some distance downstream which we name *transition point*, the boundary layer becomes turbulent. The flow-wise position of the transition point depends on the flow properties and is somewhat predictable. Further downstream, the boundary layer becomes fully turbulent. It has larger thickness than in the laminar regime, and grows at a faster rate. Like all turbulent flows, it then features strong energy dissipation and its analytical description becomes much more difficult.

The flow within the boundary layer, and the main external flow (outside of it) affect one another, but may be very different in nature. A laminar boundary layer may exist within a turbulent main flow; while turbulent boundary layers are commonplace in laminar flows.

10.2.2 Why do we study the boundary layer?

Expending our energy on solving such a minuscule area of the flow may seem counter-productive, yet three great stakes are at play here:

- First, a good description allows us to **avoid having to solve the Navier-Stokes equations in the whole flow**.

Indeed, outside of the boundary layer and of the wake areas, viscous effects can be safely neglected. Fluid flow can then be described with $\rho \frac{D\vec{V}}{Dt} = -\vec{\nabla}p$ (the Euler equation, which we will introduce as eq. 11/3 in the next chapter) with acceptably low inaccuracy. As we shall soon see, this allows us to find many interesting solutions, all within reach of human comprehension and easily obtained with computers. Unfortunately, they cannot account for shear on walls.

Thus, solving the flow within the boundary layer, whether analytically or experimentally, allows us to solve the rest of the flow in a simplified manner (fig. 10.3).

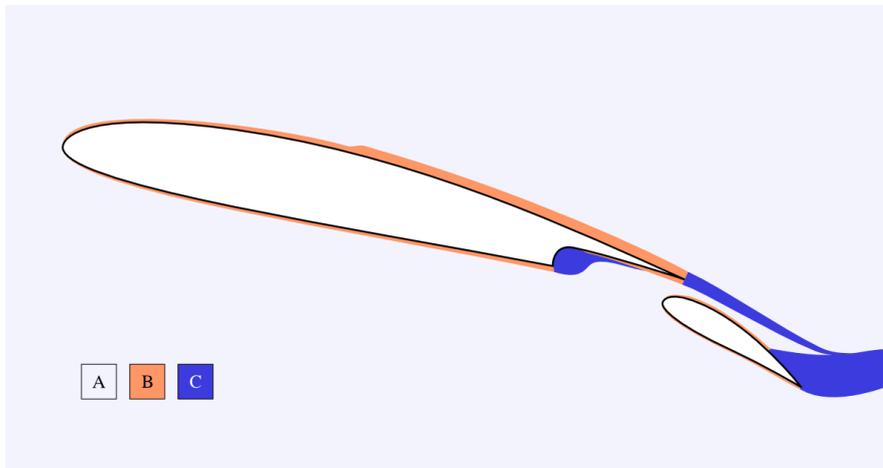


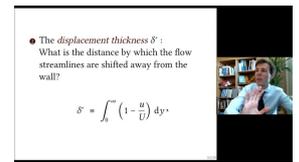
Figure 10.3: Fluid flow around a wing profile. When analyzing the flow, whether analytically or within a computational fluid dynamics (CFD) simulation, the flow domain is frequently split into three distinct areas. In the boundary layer (B), fluid flow is dominated by viscosity. Outside of the boundary layer (A), viscous effects are very small, and the flow can be approximately solved using the Euler equation. Lastly, in the turbulent wake (C), characterization of the flow is very difficult and typically necessitates experimental investigations.

Figure CC-BY-SA Olivier Cleynen

- Secondly, the boundary layer is the key to **quantifying friction**. A good resolution of the boundary layer allows us to precisely quantify the shear forces generated by a fluid on an object.
- Finally, a good understanding of the mechanisms at hand within the boundary layer allows us to predict **flow separation**, which is the divergence of streamlines relative to the object. Control of the boundary layer is key to ensuring that a flow will follow a desired trajectory!

10.2.3 Characterization of the boundary layer

Three different parameters are typically used to quantify how thick a boundary layer is at any given position.



Video: the basics of the boundary layer

by Olivier Cleynen (CC-BY)
<https://youtu.be/cgWIYSnvlEg>

The first is the *thickness* δ ,

$$\delta \equiv y|_{u=0.99U} \quad (10/1)$$

which is, as we have seen above, equal to the distance away from the wall where the speed u is 99 % of U .

The second is the *displacement thickness* δ^* , which corresponds to the vertical distance by which streamlines outside of the boundary layer have been displaced. This vertical “shifting” of the flow occurs because the inner fluid is slowed down near the wall, creating some blockage for the outer flow, which then proceeds to avoid it partially by deviating outwards (fig. 10.4).

Integral analysis performed on a control volume enclosing the boundary layer (as for example in exercise 3.8 from chapter 3) allows us to quantify the displacement thickness as:

$$\delta^* \equiv \int_0^\infty \left(1 - \frac{u}{U}\right) dy \quad (10/2)$$

In practice, this integral can be calculated on a finite interval (instead of using the ∞ limit), as long as the upper limit exceeds the boundary layer thickness.

The third and last parameter is the *momentum thickness* δ^{**} (sometimes written θ) which is equal to the thickness of a corresponding layer of fluid at velocity U which would possess the same amount of momentum as the boundary layer. The momentum thickness can be thought of as the thickness of the fluid that would need to be entirely stopped (for example by pumping it outside of the main flow) in order to generate the same drag as the local boundary layer.

A review of the experience we gathered in chapter 3 while solving problems 3.7 to 3.9 p. 67 allows us to quantify the momentum thickness as:

$$\delta^{**} \equiv \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy \quad (10/3)$$

with the same remark regarding the upper limit. The momentum thickness, in particular when compared to the displacement thickness, is an important parameter used in prediction models for boundary layer separation.

Once these three thicknesses have been quantified, we are generally looking for a quantification of the shear term τ_{wall} . Since we are working with the

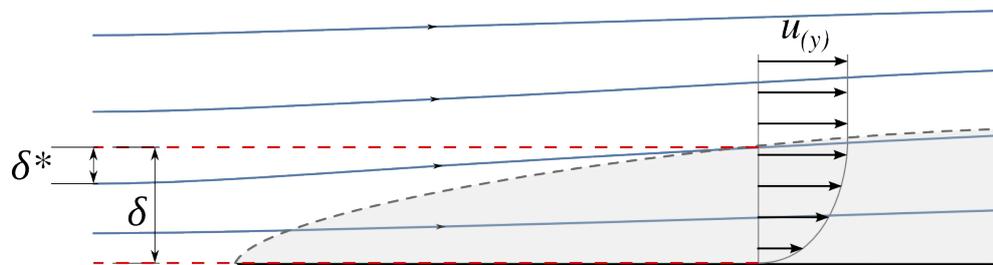


Figure 10.4: Thickness δ and displacement thickness δ^* of a boundary layer. It is important to understand that the boundary layer is not a closed zone: streamlines (drawn blue) penetrate it and the vertical velocity v , although very small compared to u , is not zero.

Figure CC-BY-SA Olivier Cleynen & Commons User:Flanker

hypothesis that the fluid is Newtonian, we merely have to know $u(y)$ to quantify shear, according to equation 5/22 which we wrote way back p. 97:

$$\tau_{\text{wall } yx} = \mu \frac{\partial u}{\partial y} \quad (10/4)$$

In a boundary layer, the shear τ_{wall} will decrease with longitudinal distance x , because the velocity gradient above it also decreases. Consequently, τ_{wall} will become a function of x , so that the entire shear force will be obtained by integration (reusing eq. 5/3 p. 92):

$$F_{\text{shear } yx} = \int_S \tau_{\text{wall } yx} dx dz \quad (10/5)$$

As we have seen in the previous chapter, fluid dynamicists like to quantify phenomena with non-dimensional parameters. The wall shear exerted by the boundary layer is typically non-dimensionalized with the *shear coefficient* c_f ,

$$c_{f(x)} \equiv \frac{\tau_{\text{wall}}}{\frac{1}{2}\rho U^2} \quad (10/6)$$

where U is the outer-layer (free-stream) velocity.

The shear coefficient, just like the shear, remains a function of the flow-wise distance x .

10.3 Laminar boundary layers

10.3.1 Governing equations

What is happening inside a laminar, steady boundary layer? We begin by writing out the Navier-Stokes for incompressible isothermal flow in two Cartesian coordinates (eqs. 6/42 p. 125):

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = \rho g_x - \frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{(\partial x)^2} + \frac{\partial^2 u}{(\partial y)^2} \right] \quad (10/7)$$

$$\rho \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = \rho g_y - \frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v}{(\partial x)^2} + \frac{\partial^2 v}{(\partial y)^2} \right] \quad (10/8)$$

Building from these two equations, we are going to add three simplifications, which are hypotheses based on experimental observation of fluid flow in boundary layers:

1. Gravity plays a negligible role;
2. The component of velocity perpendicular to the wall (in our convention, v) is very small ($v \ll u$).
Thus, its stream-wise spatial variations can also be neglected: $\partial v / \partial x \approx 0$ and $\partial^2 v / (\partial x)^2 \approx 0$. The same goes for the derivatives in the y -direction: $\frac{\partial v}{\partial y} \approx 0$ and $\partial^2 v / (\partial y)^2 \approx 0$.
3. The component of velocity parallel to the wall (in our convention, u) varies much more strongly in the y -direction than in the x -direction: $\partial^2 u / (\partial x)^2 \ll \partial^2 u / (\partial y)^2$.

With all of these simplifications, equation 10/8 shrinks down to

$$\frac{\partial p}{\partial y} \approx 0 \quad (10/9)$$

which tells us that pressure is a function of x only ($\partial p / \partial x = dp / dx$).

We now turn to equation 10/7, first to obtain an expression for pressure by applying it outside of the boundary layer where $u = U$:

$$\frac{dp}{dx} = -\rho U \frac{dU}{dx} \quad (10/10)$$

and secondly to obtain an expression for the velocity profile:

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{\partial^2 u}{(\partial y)^2} \\ &= U \frac{dU}{dx} + \frac{\mu}{\rho} \frac{\partial^2 u}{(\partial y)^2} \end{aligned} \quad (10/11)$$

Thus, the velocity field $\vec{V} = (u; v) = f(x, y)$ in a steady laminar boundary layer is driven by the two following equations: a balance of momentum, and a balance of mass:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \frac{\mu}{\rho} \frac{\partial^2 u}{(\partial y)^2} \quad (10/12)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (10/13)$$

The main unknown in this system is the longitudinal speed profile across the layer, $u_{(x,y)}$. Unfortunately, over a century after it has been written, we still have not found an analytical solution to it.

10.3.2 Blasius' solution

Heinrich Blasius undertook a PhD thesis under the guidance of Prandtl, in which he focused on the characterization of laminar boundary layers. As part of his work, he showed that the geometry of the velocity profile (i.e. the velocity distribution) within such a layer is *always the same*, and that regardless of the flow velocity or the position, u can be simply expressed as a function of non-dimensionalized distance away from the wall termed η :

$$\eta \equiv y \sqrt{\frac{\rho U}{\mu x}} \quad (10/14)$$

Blasius was able to show that u is a function such that $u/U = f'_{(\eta)}$, with $f''' + \frac{1}{2}ff'' = 0$. Unfortunately, no known analytical solution to this equation is known. However, it has now long been possible to obtain numerical values for f' at selected positions η . Those are plotted in fig. 10.5.

Based on this work, it can be shown that for a laminar boundary layer flowing along a smooth wall, the four parameters of interest for the engineer are

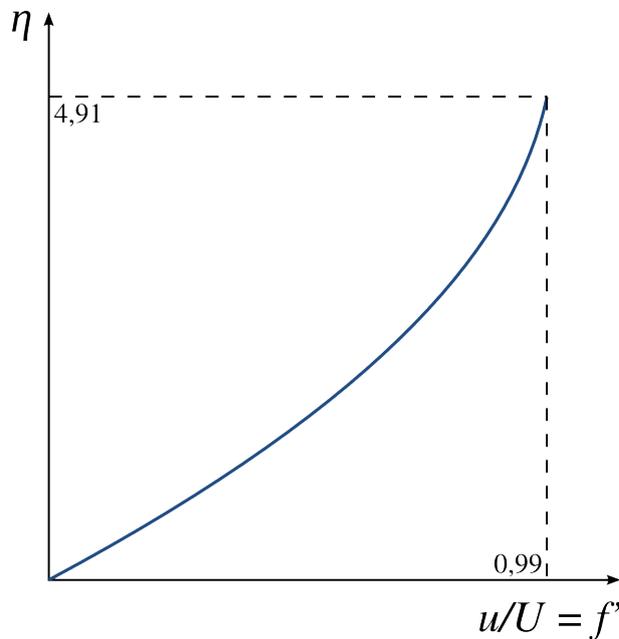


Figure 10.5: The velocity profile obtained by Blasius (an exact solution to the Navier-Stokes equations simplified with laminar boundary-layer hypothesis).

Figure CC-0 Olivier Cleynen

simply functions of the distance-based Reynolds number $[\text{Re}]_x$:

$$\frac{\delta}{x} = \frac{4,91}{\sqrt{[\text{Re}]_x}} \quad (10/15)$$

$$\frac{\delta^*}{x} = \frac{1,72}{\sqrt{[\text{Re}]_x}} \quad (10/16)$$

$$\frac{\delta^{**}}{x} = \frac{0,664}{\sqrt{[\text{Re}]_x}} \quad (10/17)$$

$$c_{f(x)} = \frac{0,664}{\sqrt{[\text{Re}]_x}} \quad (10/18)$$

10.4 Boundary layer transition

After it has traveled a certain length along the wall, the boundary layer becomes very unstable and it transits rapidly from a laminar to a turbulent regime (fig. 10.6). We have already described the characteristics of turbulence in broadly in chapter 7 (*Pipe flows*) and more extensively in chapter 9 (*Dealing with turbulence*); they apply to turbulence within the boundary layer. It is worth reminding ourselves that the boundary layer may be turbulent in a globally laminar flow (e.g. around an aircraft in flight, the boundary layer is turbulent, but the main flow is laminar). Here, we refer to the regime of the boundary layer only, not the outer flow.

We observe that the distance $x_{\text{transition}}$ at which the boundary layer changes regime is reduced when the velocity is increased, or when the viscosity is decreased. In practice this distance depends on the distance-based Reynolds number $[\text{Re}]_x \equiv \rho U x / \mu$. The most commonly accepted prediction for the transition position is:

$$[\text{Re}]_{x \text{ transition}} \approx 5 \cdot 10^5 \quad (10/19)$$

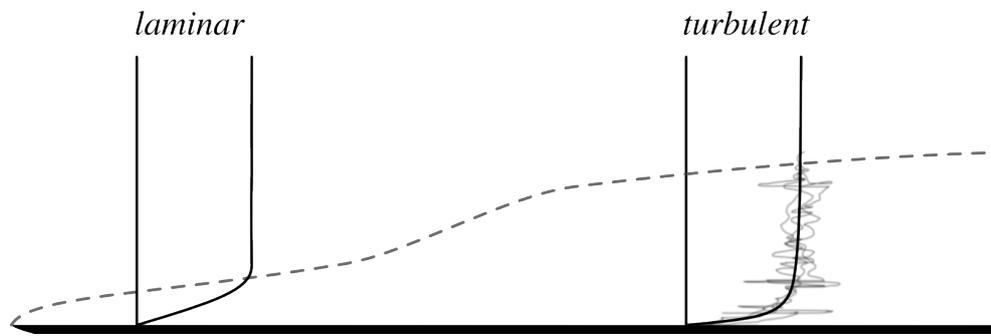
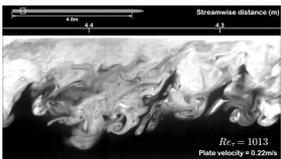


Figure 10.6: Transition of a boundary layer from laminar to turbulent regime.

Figure CC-BY Olivier Cleynen

Transition can be generated earlier if the surface roughness is increased, or if obstacles (e.g. turbulators, vortex generators, trip wires) are positioned within the boundary layer. Conversely, a very smooth surface and a very steady, uniform incoming flow will result in delayed transition.

10.5 Turbulent boundary layers



Video: visualizing the turbulent boundary layer on flat body moved in a stationary tank

by Lee, Kwon, Hutchins, and Monty [28] (STYL)
<https://youtu.be/e1TbKLIDWys>

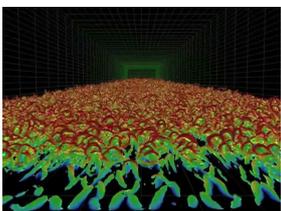
The extensive description of turbulent flows remains an unsolved problem. As we have seen in chapter 9 (*Dealing with turbulence*), by contrast with laminar counterparts, turbulent flows result in

- increased mass, energy and momentum exchange;
- increased losses to friction;
- apparently chaotic internal movements.

Instead of resolving the entire time-dependent flow in the boundary layer, we satisfy ourselves with describing the average component of the longitudinal speed, \bar{u} . A widely-accepted velocity model is:

$$\frac{\bar{u}}{U} \approx \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \quad (10/20)$$

for turbulent boundary layer flow over a smooth surface.



Video: a look inside a turbulent boundary layer, with a completely resolved (DNS) fluid flow simulation

by Linné Flow Centre & SERC KTH [26, 27] (STYL)
<https://youtu.be/4KeaAhVoPIw>

This profile has a much flatter geometry near the wall than its laminar counterpart (fig. 10.7).

In the same way that we have worked with the laminar boundary layer profiles, we can derive models for our characteristics of interest from this velocity profile:

$$\frac{\delta}{x} \approx \frac{0,16}{[\text{Re}]_x^{\frac{1}{7}}} \quad (10/21)$$

$$\frac{\delta^*}{x} \approx \frac{0,02}{[\text{Re}]_x^{\frac{1}{7}}} \quad (10/22)$$

$$\frac{\delta^{**}}{x} \approx \frac{0,016}{[\text{Re}]_x^{\frac{1}{7}}} \quad (10/23)$$

$$c_{f(x)} \approx \frac{0,027}{[\text{Re}]_x^{\frac{1}{7}}} \quad (10/24)$$

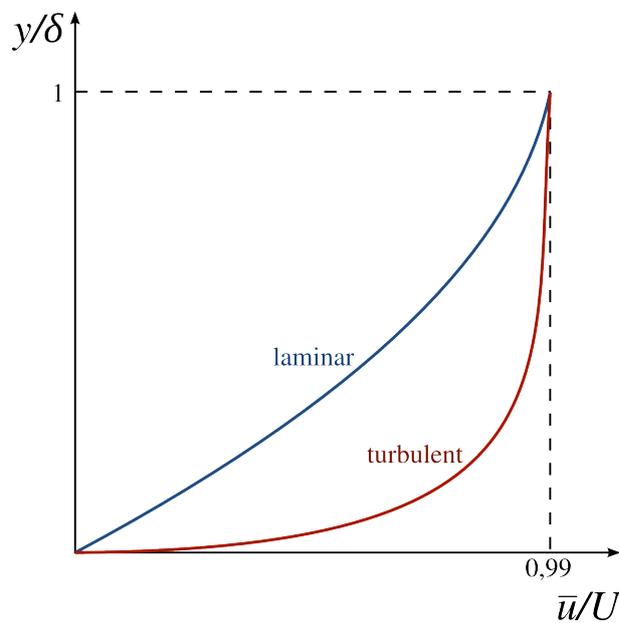


Figure 10.7: Comparison of laminar and turbulent boundary layer profiles, both scaled to the same size. It is important to remember that in practice turbulent boundary layers are much thicker than laminar ones, and that \bar{u} is only a time-average velocity.

Figure CC-0 Olivier Cleynen

10.6 Flow separation

Under certain conditions, fluid flow separates from the wall. The boundary layer then disintegrates and we observe the appearance of a turbulent wake near the wall. Separation is often an undesirable phenomenon in fluid mechanics: it may be thought of as the point where we fail to impart a desired trajectory to the fluid.

When the main flow speed U along the wall is varied, we observe that the geometry of the boundary layer changes. The greater the longitudinal speed gradient ($dU/dx > 0$), and the flatter the profile becomes. Conversely, when the longitudinal speed gradient is negative, the boundary layer velocity profile straightens up. When it becomes perfectly vertical at the wall, it is such that streamlines separate from the wall: this is called *separation*^w (fig. 10.8).

The occurrence of separation can be predicted if we have a robust model for the velocity profile inside the boundary layer. For this, we go back to fundamentals, stating that **at the separation point, the shear effort on the surface must be zero**:

$$\tau_{\text{wall at separation}} = 0 = \mu \left(\frac{\partial u}{\partial y} \right)_{@y=0} \quad (10/25)$$

At the wall surface ($u = 0$ and $v = 0$), equation 10/12 p. 204 becomes:

$$\mu \left(\frac{\partial^2 u}{(\partial y)^2} \right)_{@y=0} = \frac{dp}{dx} = -\rho U \frac{dU}{dx} \quad (10/26)$$

Thus, as we progressively increase the term dp/dx , the term $\partial^2 u/(\partial y)^2$ reaches higher (positive) values on the wall surface. Nevertheless, we know



Video: flow separation: what it is, how to predict and avoid it
by Olivier Cleynen (CC-BY)
<https://youtu.be/zB5g4aB1Sus>

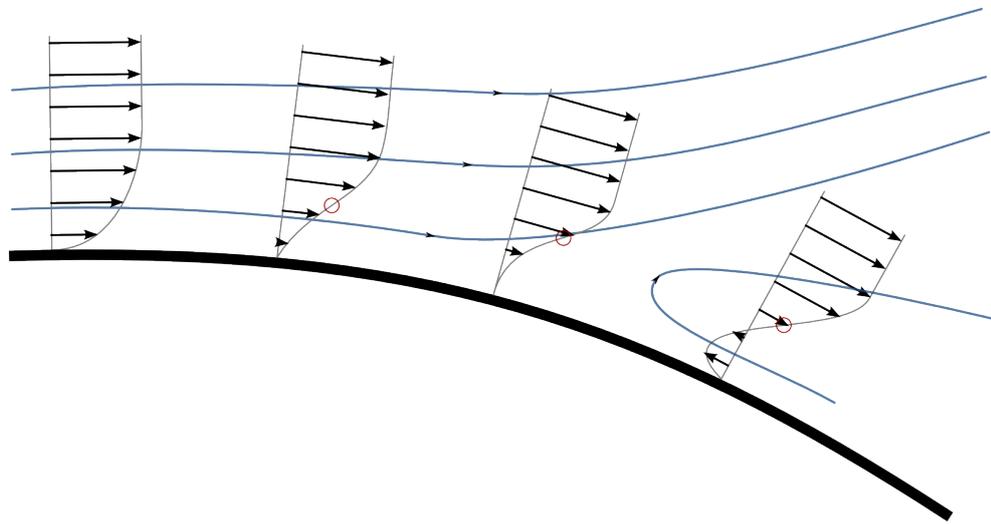


Figure 10.8: Separation of the boundary layer. The main flow is from left to right, and flowing towards a region of increasing pressure. For clarity, the y -scale is greatly exaggerated.

Figure CC-BY Olivier Cleynen

that it *must* take a negative value at the exterior boundary of the boundary layer. Therefore, it must change sign somewhere in the boundary. This point where $\partial^2 u / (\partial y)^2$ changes sign is called *inflection point*.

The existence of the inflection point within the boundary layer tells us that at the wall ($y = 0$) the term $\partial u / \partial y$ tends towards ever smaller values. Given enough distance x , it will reach zero value, and the boundary layer will separate (fig. 10.9). Therefore, the longitudinal pressure gradient, which in practice determines the longitudinal velocity gradient, is the key factor in the analytical prediction of separation.

We shall remember two crucial points regarding the separation of boundary layers:

1. **Separation occurs in the presence of a positive pressure gradient**, which is sometimes named *adverse pressure gradient*.

Separation points along a wall (e.g. a car bodywork, an aircraft wing, rooftops, mountains) are always situated in regions where pressure increases (positive dp/dx or negative dU/dx). If pressure remains constant, or if it decreases, then the boundary layer cannot separate.

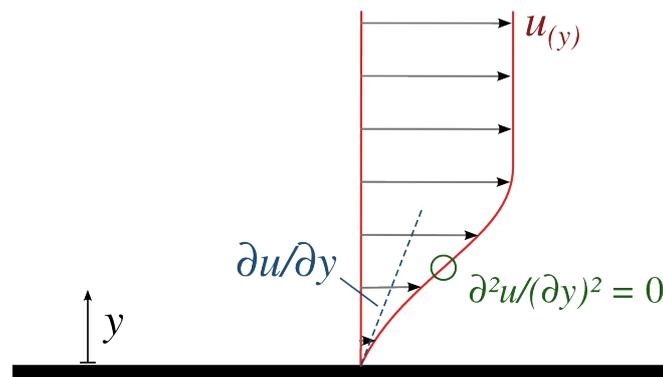


Figure 10.9: The inflection point within a boundary layer about to separate.

Figure CC-BY Olivier Cleynen

2. **Laminar boundary layers are much more sensitive to separation** than turbulent boundary layers (fig. 10.10).

A widely-used technique to reduce or delay the occurrence of separation is to make boundary layers turbulent, using low-height artificial obstacles positioned in the flow. By doing so, we increase shear-based friction (which increases with turbulence) as a trade-off for better resistance to stall.

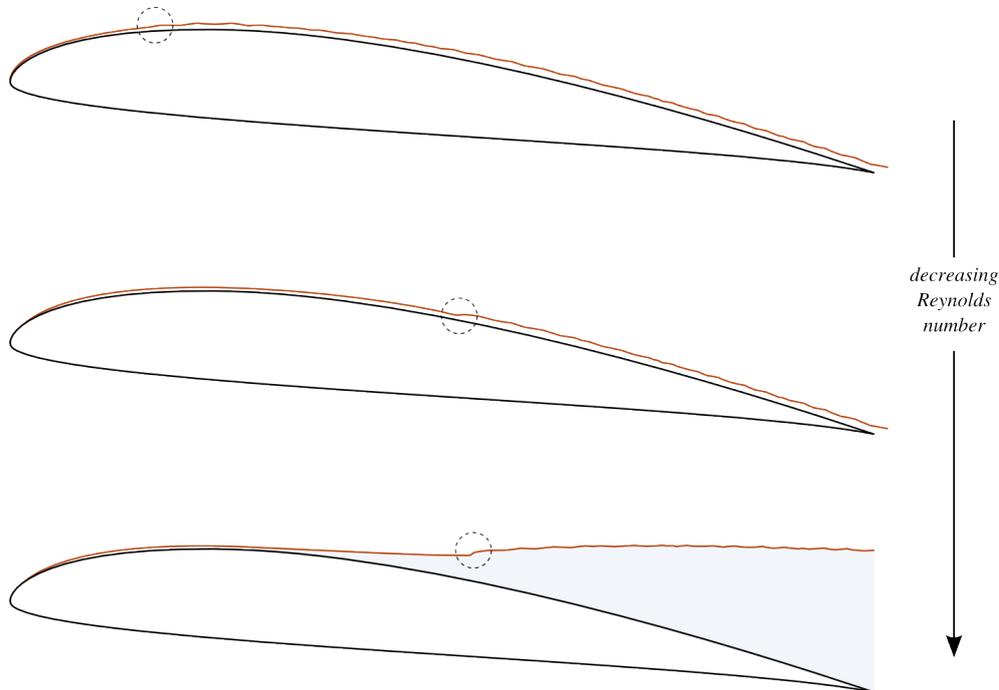


Figure 10.10: The effect of decreasing Reynolds number on flow attachment over an airfoil at constant angle of attack, with the transition point highlighted. Laminar boundary layers are much more prone to separation than turbulent boundary layers.

Figure CC-BY-SA Olivier Cleynen, based on Barlow & Pope 1999 [11]

Predicting in practice the position of a separation point is difficult, because an intimate knowledge of the boundary layer profile and of the (external-flow-generated) pressure field are required – and as the flow separates, these are no longer independent. Resorting to experimental measurements, in this case, is often a wise idea!



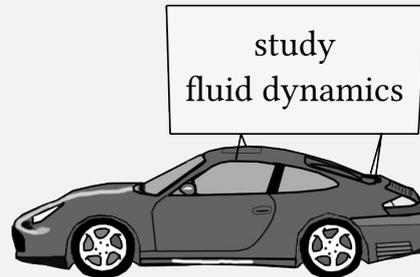
Video: a football with a rough edge on one side will see turbulent boundary layer on that side and laminar boundary layer on the other. When the laminar layer separates before the turbulent one, the main flow around the ball becomes asymmetric and deviates the ball sideways, a phenomenon that can be exploited to perform impressive tricks.

by freekikerz (STVL)
<https://youtu.be/rzRuZNvkMbc>

10.7 Solved problems

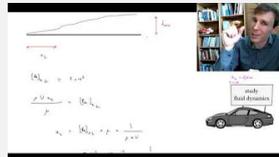
Advertising board on a car

A successful fluid dynamics professor advertises for their course using a board above their car. They drive at 10 m s^{-1} ; the board is 3 m long and 1,5 m high.



Drawing CC-BY-SA by Imad Kharkouk

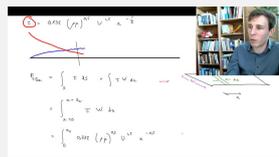
Will the boundary layer on the board become turbulent? How thick will it become?



See this solution worked out step by step on YouTube
<https://youtu.be/ML5dqjHeqPE> (CC-BY Olivier Cleynen)

Shear force on a board (laminar part)

In the example above, what is the shear force on the laminar layer part of the board?



See this solution worked out step by step on YouTube
<https://youtu.be/xxoqwzihRMc> (CC-BY Olivier Cleynen)

Note: Unfortunately Olivier made an error in this video: the final expression is correct, but improperly calculated. The correct result is 0,107 N. Many thanks to the students who double-checked and reported the problem!

Shear force on a board (turbulent part)

In the example above, what is the shear force on the turbulent layer part of the board? And what would be the power lost to friction on the entire board?



See this solution worked out step by step on YouTube
https://youtu.be/6i_yu1BKkVY (CC-BY Olivier Cleynen)

Problem sheet 10: Flow near walls

last edited July 9, 2020

by Olivier Cleynen – <https://fluidmech.ninja/>

Except otherwise indicated, assume that:

The atmosphere has $p_{\text{atm.}} = 1 \text{ bar}$; $\rho_{\text{atm.}} = 1,225 \text{ kg m}^{-3}$; $T_{\text{atm.}} = 11,3 \text{ }^\circ\text{C}$; $\mu_{\text{atm.}} = 1,5 \cdot 10^{-5} \text{ Pa s}$

Air behaves as a perfect gas: $R_{\text{air}}=287 \text{ J kg}^{-1} \text{ K}^{-1}$; $\gamma_{\text{air}}=1,4$; $c_{p \text{ air}}=1 005 \text{ J kg}^{-1} \text{ K}^{-1}$; $c_{v \text{ air}}=718 \text{ J kg}^{-1} \text{ K}^{-1}$

Liquid water is incompressible: $\rho_{\text{water}} = 1 000 \text{ kg m}^{-3}$, $c_{p \text{ water}} = 4 180 \text{ J kg}^{-1} \text{ K}^{-1}$

In boundary layer flow, we assume that transition occurs at $[\text{Re}]_x \gtrsim 5 \cdot 10^5$.

The wall shear coefficient c_f , a function of distance x , is defined based on the free-stream flow velocity U :

$$c_{f(x)} \equiv \frac{\tau_{\text{wall}}}{\frac{1}{2}\rho U^2} \quad (10/6)$$

Exact solutions to the laminar boundary layer along a smooth surface yield:

$$\frac{\delta}{x} = \frac{4,91}{\sqrt{[\text{Re}]_x}} \quad \frac{\delta^*}{x} = \frac{1,72}{\sqrt{[\text{Re}]_x}} \quad (10/16)$$

$$\frac{\delta^{**}}{x} = \frac{0,664}{\sqrt{[\text{Re}]_x}} \quad c_{f(x)} = \frac{0,664}{\sqrt{[\text{Re}]_x}} \quad (10/18)$$

Solutions to the turbulent boundary layer along a smooth surface yield the following time-averaged characteristics:

$$\frac{\delta}{x} \approx \frac{0,16}{[\text{Re}]_x^{\frac{1}{7}}} \quad \frac{\delta^*}{x} \approx \frac{0,02}{[\text{Re}]_x^{\frac{1}{7}}} \quad (10/22)$$

$$\frac{\delta^{**}}{x} \approx \frac{0,016}{[\text{Re}]_x^{\frac{1}{7}}} \quad c_{f(x)} \approx \frac{0,027}{[\text{Re}]_x^{\frac{1}{7}}} \quad (10/24)$$

Figure 10.11 quantifies the viscosity of various fluids as a function of temperature.

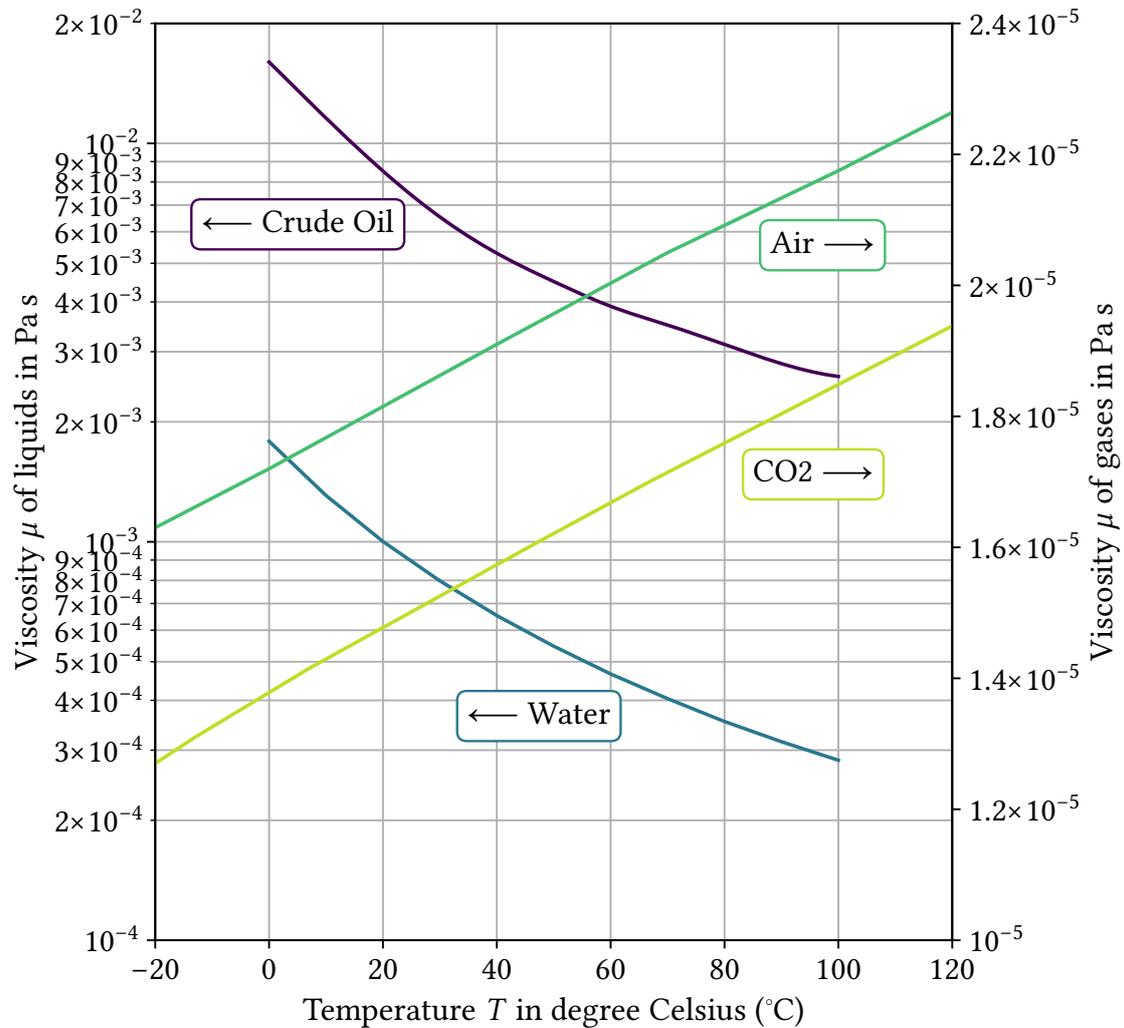


Figure 10.11: The viscosity of four fluids (crude oil, water, air, and CO₂) as a function of temperature. The scale for liquids is logarithmic and displayed on the left; the scale for gases is linear and displayed on the right.

Figure reproduced from figure 5.6 p. 99; CC-BY by Arjun Neyyathala & Olivier Cleynen

10.1 Water and air flow

White [22] E7.2

A flat plate of length 0,3 m is placed parallel to a uniform flow with speed 0,3 m s⁻¹. How thick can the boundary layer become:

- 10.1.1. if the fluid is air at 1 bar and 20 °C?
- 10.1.2. if the fluid is water at 20 °C?

10.2 Boundary layer sketches

CC-0 Olivier Cleynen

A thin and long horizontal plate is moved horizontally through a stationary fluid.

- 10.2.1. Sketch the velocity profile of the fluid:

- at the leading edge;
- at a point where the boundary layer is laminar;

- and at a point further downstream where the boundary layer is turbulent.
- 10.2.2. Draw a few streamlines, indicate the boundary layer thickness δ , and the displacement thickness δ^* .
 - 10.2.3. Explain shortly (e.g. in 30 words or less) how the transition to turbulent regime can be triggered.
 - 10.2.4. Explain shortly (e.g. in 30 words or less) how the transition to turbulent regime could instead be delayed.

10.3 Shear force due to boundary layer

White [22] E7.3

A thin and smooth plate of dimensions $0,5 \times 3$ m is placed with a zero angle of attack in a flow incoming at $1,25 \text{ m s}^{-1}$, as shown in fig. 10.12.

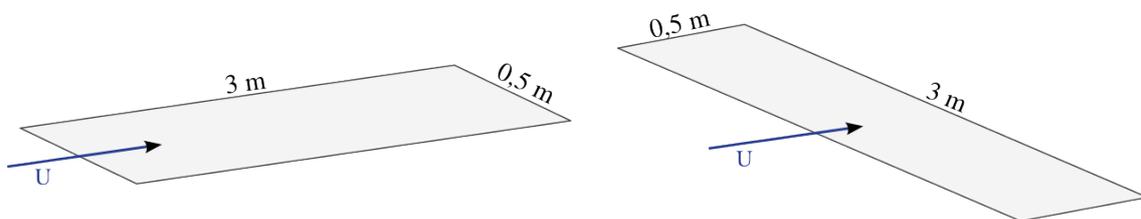


Figure 10.12: A thin plate positioned parallel to an incoming uniform flow. Two configurations are studied in this exercise.

Figure CC-0 Olivier Cleynen

- 10.3.1. What is the shear force exerted on the top surface of the plate for each of the two configurations shown in fig. 10.12, when the fluid is air of viscosity $\mu_{\text{air}} = 1,5 \cdot 10^{-5} \text{ Pa s}$?
- 10.3.2. What are the shear forces when the fluid is water of viscosity $\mu_{\text{water}} = 1 \cdot 10^{-3} \text{ Pa s}$?
- 10.3.3. [difficult question] How would these shear efforts evolve if the plate was tilted with an angle of 20° relative to the flow?

10.4 Wright Flyer I

CC-0 Olivier Cleynen

The **Wright Flyer I**, the first airplane capable of sustained controlled flight (1903), was a biplane with a 12 m wingspan (fig. 10.13). It had two wings of chord length 1,98 m stacked one on top of the other. The wing profile was extremely thin and it could only fly at very low angles of attack. Its flight speed was approximately 40 km h^{-1} .

- 10.4.1. If the flow over the wings can be treated as if they were flat plates, what is the power necessary to compensate the shear exerted by the airflow on the wings during flight?
- 10.4.2. Which other forms of drag would also be found on the aircraft? (give a brief answer, e.g. in 30 words or less)



Figure 10.13: The Wright Flyer I, first modern airplane in history. Built with meticulous care and impeccable engineering methodology by two bicycle makers, it made history in December 1903.

Photo by Orville Wright, 1908 (public domain)

10.5 Power lost to shear on an airliner fuselage CC-0 Olivier Cleynen

An Airbus A340-600 (fig. 10.14) is cruising at $[Ma] = 0,82$ at an altitude of 10 000 m (where the air has viscosity $1,457 \cdot 10^{-5} \text{ N s m}^{-2}$, temperature 220 K, density $0,4 \text{ kg m}^{-3}$).



Figure 10.14: The Airbus A340-600, a large airliner first flown in 2001.

Photo CC-BY-SA by Iberia Airlines (retouched)

The cylindrical part of the fuselage has diameter 5,6 m and length 65 m.

- 10.5.1. What is approximately the maximum boundary layer thickness around the fuselage?
- 10.5.2. What is approximately the average shear applying on the fuselage skin?
- 10.5.3. Estimate the power dissipated to friction on the cylindrical part of the fuselage.
- 10.5.4. In practice, in which circumstances could flow separation occur on the fuselage skin? (give a brief answer, e.g. in 30 words or less)

10.6 Laminar wing profile non-examinable. Based on a diagram from Bertin et al. 2010 [24]

The characteristics of a so-called “laminar” wing profile are compared in figs. 10.15 to 10.17 with those of an ordinary profile.

On the graph representing the pressure coefficient $C_p \equiv \frac{p - p_\infty}{\frac{1}{2} \rho V^2}$, identify the curve corresponding to each profile.

What advantages and disadvantages does the laminar wing profile have, and how can they be explained? In which applications will it be most useful?

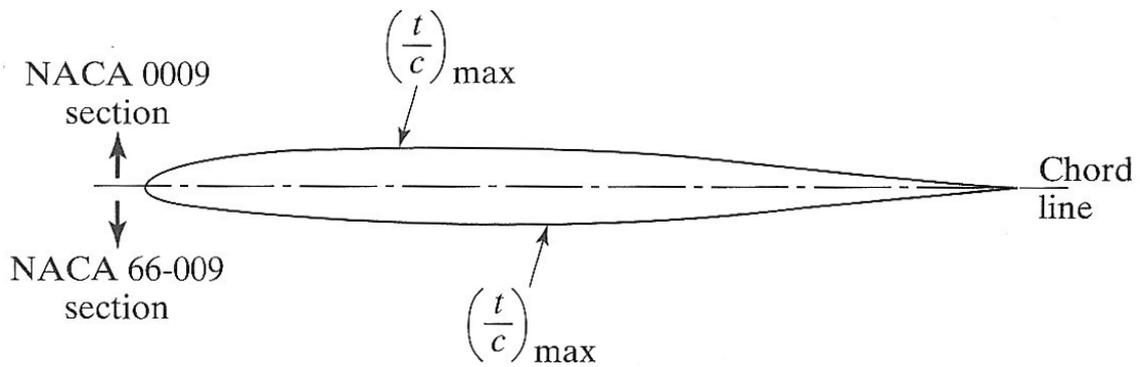


Figure 10.15: Comparison of the thickness distribution of two uncambered wing profiles: an ordinary medium-speed NACA 0009 profile, and a “laminar” NACA 66-009 profile.

Figure © Bertin & Cummings 2010 [24]

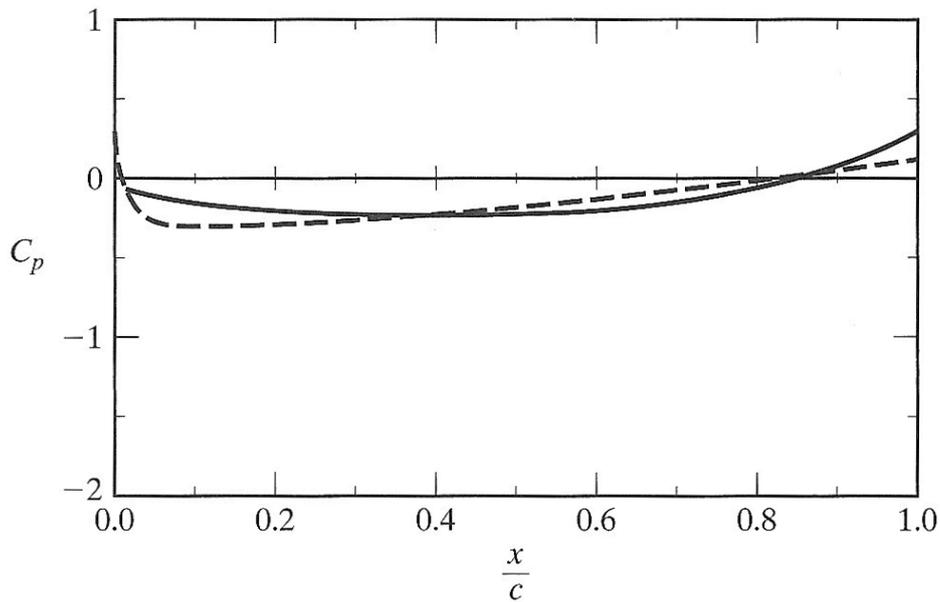


Figure 10.16: Static pressure distribution (represented as the local non-dimensional *pressure coefficient* $C_p \equiv \frac{p-p_\infty}{\frac{1}{2}\rho V^2}$) as a function of distance x (non-dimensionalized with the chord c) over the surface of the two airfoils shown in fig. 10.15.

Figure © Bertin & Cummings 2010 [24]

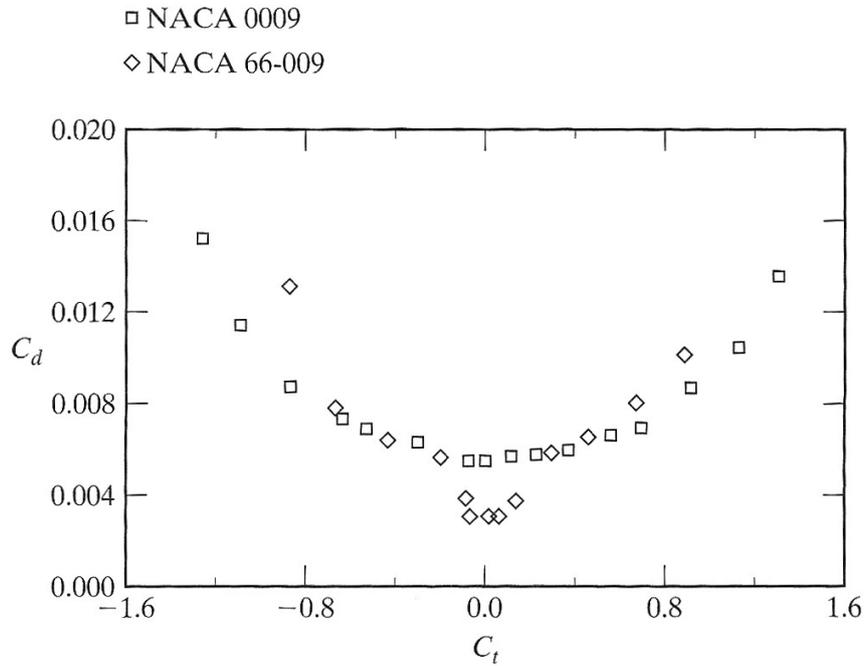


Figure 10.17: Values of the section drag coefficient $C_d \equiv \frac{d}{\frac{1}{2}c\rho V^2}$ as a function of the section lift coefficient $C_l \equiv \frac{l}{\frac{1}{2}c\rho V^2}$ for both airfoils presented in fig. 10.15.

Figure © Bertin & Cummings 2010 [24], based on data by Abbott & Von Doenhoff 1949 [1]

10.7 Separation mechanism

non-examinable, CC-0 Olivier Cleynen

Sketch the velocity profile of a laminar or turbulent boundary layer shortly upstream of, and at a separation point.

The two equations below describe flow in laminar boundary layer:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \frac{\mu}{\rho} \frac{\partial^2 u}{(\partial y)^2} \quad (10/12)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (10/13)$$

Identify these two equations, list the conditions in which they apply, and explain shortly (e.g. in 30 words or less) why a boundary layer cannot separate when a favorable pressure gradient is applied along the wall.

Answers

- 10.1** 1) At trailing edge $[\text{Re}]_x = 5\,348$ thus the layer is laminar everywhere. δ will grow from 0 to 2,01 cm (eq. 10/15 p. 205);
2) For water: $\delta_{\text{trailing edge}} = 4,91$ mm.
- 10.2** 1) See fig. 10.6 p. 206. At the leading-edge the velocity is uniform. Note that the y -direction is greatly exaggerated, and that the outer velocity U is identical for both regimes;
2) See fig. 10.4 p. 202. Note that streamlines penetrate the boundary layer;
3) and 4) See §10.4 p. 205.
- 10.3** $x_{\text{transition, air}} = 4,898$ m and $x_{\text{transition, water}} = 0,4$ m. In a laminar boundary layer, inserting equation 10/18 into equation 10/6 into equation 10/5 yields
$$F_\tau = 0,664LU^{1,5} \sqrt{\rho\mu} \left[\sqrt{x} \right]_0^{x_{\text{transition}}}.$$

In a turbulent boundary layer, we use equation 10/24 instead and get
$$F_\tau = 0,01575L\rho^{\frac{6}{7}}U^{\frac{13}{7}}\mu^{\frac{1}{7}} \left[x^{\frac{6}{7}} \right]_{x_{\text{transition}}}^{x_{\text{trailing edge}}}.$$
 These expressions allow the calculation of the forces below, for the top surface of the plate:
- 1) (air) First case: $F = 3,445 \cdot 10^{-3}$ N; second case $F = 8,438 \cdot 10^{-3}$ N (who would have thought eh?);
2) (water) First case: $F = 3,7849$ N; second case $F = 2,7156$ N.
- 10.4** Using the expressions developed in exercise 10.3, $\dot{W}_{\text{friction}} \approx 255$ W.
- 10.5** 1) $x_{\text{transition}} = 7,47$ cm (the laminar part is negligible). With the equations developed in exercise 7.3, we get $F = 24,979$ kN and $\dot{W} = 6,09$ MW. Quite a jump from the Wright Flyer I!
2) When the longitudinal pressure gradient is zero, the boundary layer cannot separate. Thus separation from the fuselage skin can only happen if the fuselage is flown at an angle relative to the flight direction (e.g. during a low-speed maneuver).

