Fluid Dynamics
Chapter 1 – Basic flow quantities

last edited April 18, 2021
by Olivier Cleynen — https://fluidmech.ninja/

These notes are based on textbooks by White [22], Çengel & al.[25], Munson & al.[29], and de Nevers [17].

1.1 Concept of a fluid

We call *fluid*[W] a type of matter which is continuously deformable, and which spontaneously tends to adapt its shape to its container by occupying all of the space made available to it.

1.2 Fluid dynamics

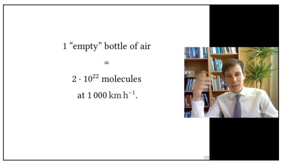
1.2.1 Solution of a flow

Fluid dynamics (or generally, fluid mechanics) is the study of the movement of fluids. The most common type of problem in this discipline is the search for a complete description of the fluid flow around or through a solid object. This problem is solved when the entire set of velocities of fluid particles has been described. This set of velocities, which is often a function of time, can be described either as a set of discrete values ("pixelized" data) or as a mathematical function; it is called the *flow solution*.

If the solution is known, the shear and pressure efforts generated on the surface of the object can be calculated. Other quantities, such as the force and moments applying on the object, or the fluid's energy gains or losses, can also be calculated.

1.2.2 Modeling of fluids

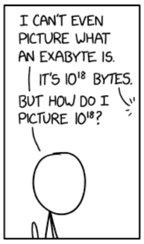
Like all matter, fluids are made of discrete, solid molecules. However, in fluid mechanics, we work at the macroscopic scale: at that scale, matter can be treated like a *continuum*, in which all physical properties of interest can be continuously differentiated.



Video: why not to calculate the movement of molecules
by Olivier Cleynen (CC-BY)
https://youtu.be/37K7GxYnebk

There are about $2 \cdot 10^{22}$ molecules in the air within an "empty" 1-liter bottle at ambient temperature and pressure. Even when the air within the bottle is completely still, these molecules are constantly colliding with each other and with the bottle walls; on average, their speed is equal to the speed of sound: approximately 1 000 km/h.

Despite the complexity of individual molecule movements, even the most turbulent flows can be accurately described and solved by considering the velocities of *groups* of several millions of molecules collectively, which we name *fluid particles*.[W] By doing so, we never find out the velocity of individual molecules: instead, those are averaged in space and time and result in much simpler and smoother trajectories, which are those we can observe with macroscopic instruments such as video cameras and pressure probes.



XKCD #2283: how (not) to picture big numbers
by Randall Munroe (CC-BY-NC)
https://xkcd.com/2283

Our selection of an appropriate fluid particle size (in effect defining the lower boundary of the *macroscopic scale*), is illustrated in fig. 1.1. We choose to reduce our volume of study to the smallest possible size before the effect of individual molecules becomes meaningful.

Adopting this point of view, which is named the *continuum abstraction*, is not a trivial decision, because the physical laws which determine the behavior of molecules are very different from those which determine the behavior of elements of fluid. For example, in fluid mechanics we never consider any inter-element attraction or repulsion forces; while new forces "appear" due to pressure or shear effects that do not exist at a molecular level.

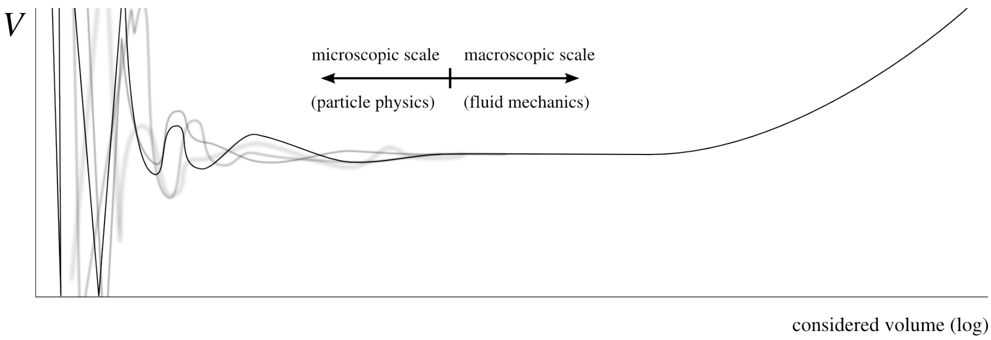


Figure 1.1: Measurement of the average value of a property (here, velocity V; but it could be pressure, or temperature) inside a given volume. As the volume shrinks towards zero, the fluid can no longer be treated as a continuum; and property measurements will oscillate wildly.

Figure CC-BY-SA *Olivier Cleynen*

A direct benefit of the continuum abstraction is that the mathematical complexity of our problems is greatly simplified. Finding the solution for the bottle of "still air" mentioned above, for example, requires only a single equation (eq. 4/15 p. 80) instead of a system of $2 \cdot 10^{22}$ equations with $2 \cdot 10^{22}$ unknowns (all leading up to $\vec{V}_{\text{average},x,y,z,t} = \vec{0}$!).

Another consequence is that we cannot treat a fluid as if it were a mere set of marbles with no interaction which would hit objects as they move by. Instead we must think of a fluid –even a low-density fluid such as atmospheric air– as an infinitely-flexible medium able to fill in almost instantly all of the space made available to it.

1.2.3 Theory, numerics, and experiment

Today, fluid dynamicists are typically specializing in any one of three sub-disciplines:

SMBC #2010-08-29: cooperation between theoretical and experimental scientists
by Zach Weinersmith
https://www.smbc-comics.com/comic/2010-08-29

Analytical fluid mechanics which is the main focus of these lectures and which consists in predicting fluid flows mathematically. As we shall see, it is only able to provide (exact) solutions for very simple flows. In fluid mechanics, theory nevertheless allows us to understand the mechanisms of complex fluid phenomena, describe scale effects, and predict forces associated with given fluid flows;

Numerical fluid mechanics also called *Computational Fluid Dynamics*[W] or CFD, which consists in solving problems using very large numbers of discrete values. Initiated as a research topic in the 1970s, CFD is now omnipresent in the industry; it allows for excellent visualization and parametric studies of very complex flows. Nevertheless, computational solutions obtained within practical time frames are inherently approximate: they need to be challenged using analysis, and calibrated using experimental measurements;

Experimental fluid mechanics which consists in reproducing phenomena of interest within laboratory conditions and observing them using experimental techniques. A very mature branch (it first provided useful results at the end of the 19th century), it is unfortunately associated with high human, equipment and financial costs. Experimental measurements are indispensable for the validation of computational simulations; meanwhile, the design of meaningful experiments necessitates a good understanding of scale effects.

Our study of analytical fluid mechanics should therefore be a useful tool to approach the other two sub-disciplines of fluid mechanics.

1.3 Important concepts in mechanics

Mechanics in general deals with the study of forces and motion of bodies. A few concepts relevant for us are recalled here.

1.3.1 Position, velocity, acceleration

The description of the movement of bodies (without reference to the causes and effects of that movement) is called *kinematics*.[w]

The *position* in space of an object can be fully expressed using three components (one for each dimension) This is usually done with a vector (here written $\vec{x}$). If the object moves, then this vector varies with time.

The *velocity* $\vec{V}$ of the object is the rate of change in time of its position:

$$\vec{V} \equiv \frac{\mathrm{d}\vec{x}}{\mathrm{d}t} \tag{1/1}$$

The length V of the velocity vector $\vec{V}$ is measured in $\mathrm{m\,s^{-1}}$:

$$V \equiv ||\vec{V}|| \tag{1/2}$$

In this document, V always is a positive number. Its formal name is *speed*, but in practice the term *velocity* is used to designate either the vector or its length, according to context.

In order to express the velocity vector completely, three distinct values (each having positive or negative values in $\mathrm{m\,s^{-1}}$) must be expressed. In this document, this is done with different notations. In Cartesian coordinates, we have:

$$\vec{V} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

In cylindrical coordinates, we write:

$$\vec{V} = \begin{pmatrix} u_r \\ u_\theta \\ u_z \end{pmatrix}$$

The *acceleration* $\vec{a}$ of the object is the rate of change in time of its velocity:

$$\vec{a} \equiv \frac{\mathrm{d}\vec{V}}{\mathrm{d}t} \tag{1/3}$$

Acceleration is especially important in mechanics because it can be deduced from Newton's second law (see eq. 1/25 p. 20 below) if the forces applying on the object are known. Acceleration can then be integrated with respect to time to obtain velocity, which can be integrated with respect to time to obtain position.
Like velocity, acceleration has three components (each measured in $\mathrm{m\,s^{-2}}$). It shows at which rate each component of velocity is changing. It may not always point in the same direction as velocity (fig. 1.2).

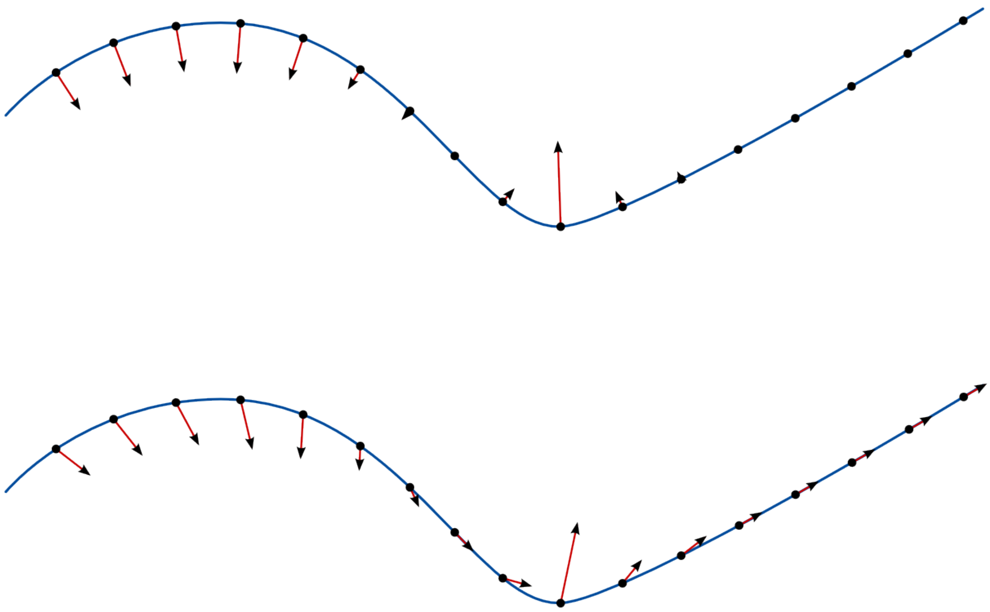


Figure 1.2: A body (black dot) is following a trajectory, plotted in blue. The acceleration vectors are plotted in red for two cases: on top, when the magnitude of its velocity remains constant, and on the bottom, when this magnitude changes continuously.

Figure CC-0 *Olivier Cleynen*

1.3.2 Forces and moments

A *force* expresses an effort exerted on a body. Force has a direction as well as a magnitude, and so it is expressed with a vector (typically noted $\vec{F}$). The three main types of forces relevant to fluid mechanics are those due to pressure, those due to shear, and those due to gravity (the force due to gravity is called *weight*).

When a force exerts at a position $\vec{r}$ away from a reference point, it exerts a torsion (or "twisting") effort named *moment*.[W] Like a force, a moment has a direction as well as a magnitude, and so is best expressed with a vector. This vector $\vec{M}$ is expressed as the cross product of the *arm* $\vec{r}$ and the force $\vec{F}$:

$$\vec{M} \equiv \vec{r} \wedge \vec{F} \tag{1/4}$$

See Appendix A2.2 p. 248 for a short briefing about the cross product of vectors.

1.3.3 Energy

Energy, measured in joules (J), is in most general terms the ability of a body to set other bodies in motion. It can be accumulated or spent by bodies in a large number of different ways. The most relevant forms of energy in fluid mechanics are:

Kinetic energy[W] noted E_k, accumulated as motion:

$$E_k \equiv \frac{1}{2} m V^2 \tag{1/5}$$

Work[w] noted W, which is energy spent on displacing an object over a distance l with a force F:

$$W \equiv \vec{F} \cdot \vec{l} \tag{1/6}$$

where W is the work (J);
$\vec{F}$ is the force (vector with magnitude in N);
and $\vec{l}$ is the movement distance (vector with magnitude in m).

See Appendix A2.1 p. 247 for a short briefing about the dot product of vectors.

Internal energy[w] noted I stored as heat within the body itself. As long as no phase changes occurs, the internal energy I of fluids is roughly proportional to their absolute temperature T.

1.4 Properties of fluids

Beyond velocity, which is the primary unknown for us in fluid mechanics, there are a few other important fluid properties.

1.4.1 Density

The density ρ (Greek letter rho) is the amount of mass per unit volume:

$$\rho \equiv \frac{m}{\mathcal{V}} \tag{1/7}$$

where ρ is the density (kg m^{-3});
m is the considered mass (kg);
and $\mathcal{V}$ is the considered volume (m^3).

Two orders of magnitude that are useful to remember: at ambient atmospheric conditions, air has a density of approximately $\rho_{\text{air}} = 1{,}2\,\text{kg m}^{-3}$; that of water is almost a thousand times greater, at $\rho_{\text{water}} = 1\,000\,\text{kg m}^{-3}$.

Advice from an expert

Never confuse pressure p, which is how hard a fluid pushes on walls, with density ρ, which is how much of the fluid there is per unit volume. Not many fluid dynamicists will confess to it, but most have, as beginner students, mixed up the two symbols once in a moment of weakness. Make sure you write those two letters very clearly, so you distinguish them easily even when stressed or distracted.

1.4.2 Phase

Fluids can be broadly classified into *phases*,[w] which are loosely-defined sets of physical behaviors. Typically one distinguishes *liquids* which are fluids with large densities on which surface tension effects play an important role, from *gases* or *vapors* which have low densities and no surface tension effects. Phase changes are often brutal (but under specific conditions can be blurred or smeared-out); they usually involve large energy transfers. The presence of multiple phases in a flow is an added layer of complexity in the description of fluid phenomena.

1.4.3 Temperature

Temperature[W] is a scalar property measured in Kelvins (an absolute scale). It represents a body's potential for receiving or providing heat and is defined, in thermodynamics, based on the transformation of heat and work.

We convert from Kelvins to degrees Celsius by subtracting 273,15 units:

$$T(^\circ\mathrm{C}) = T(\mathrm{K}) - 273{,}15 \tag{1/8}$$

Although we can "feel" temperature in daily life, it must be noted that the human body is a very poor thermometer in practice. This is because we constantly produce heat, and we infer temperature by the power our body loses or gains as heat. This power not only depends on our own body temperature (hot water "feels" hotter when we are cold), but also on the heat capacity of the fluid (cold water "feels" colder than air at the same temperature) and on the amount of convection taking place (ambient air "feels" colder on a windy day).
In spite of these impressions, the fact is that the heat capacity of fluids is extremely high ($c_{\text{air}} \approx 1\,\mathrm{kJ\,kg^{-1}\,K}$, $c_{\text{water}} \approx 4\,\mathrm{kJ\,kg^{-1}\,K}$). Unless very high velocities are attained, the temperature changes associated with fluid flow are much too small to be measurable in practice.

Advice from an expert

Never make guesses about the dynamics of a flow using your own perception of temperature. The human body constantly rejects or absorbs heat, and makes for a very bad thermometer. Wind may just *feel* cold because the air movement increases heat transfer off your skin, and not because of changes in air pressure or in density. If needed, use a real thermometer!

1.4.4 Perfect gas model

When a gas has relatively simple molecules, moderate temperature and low pressure, several of its properties can be related easily to one another with the *perfect gas model*.[W] Their absolute temperature T is then modeled as a function of their pressure p with a single, approximately constant parameter $R_{\text{specific}} \equiv p/\rho T$:

$$\frac{p}{\rho} = R_{\text{specific}}\, T \tag{1/9}$$

where R_{specific} depends on the state and nature of the gas ($\mathrm{J\,K^{-1}\,kg^{-1}}$);
and p is the pressure (Pa, see §1.5.2 further down).

Note that R_{specific} here is a *specific* gas constant (whose value depends on the gas); chemists often instead use a *universal* definition of R in $\mathrm{J\,mol^{-1}\,K^{-1}}$.

This type of model (relating temperature to pressure and density) is called an *equation of state*. When R remains approximately constant, the fluid is said to behave as a perfect gas. The properties of air can satisfactorily be predicted using this model. Other models exist[W] which predict the properties of gases over larger property ranges, at the cost of increased mathematical complexity.

Many fluids, especially liquids, do not follow this equation and their temperature must be determined in another way, most often with the help of laboratory measurements.

1.4.5 Speed of sound

An important property of fluids is the speed at which pressure changes can travel within the fluid (these pressure changes may for example be caused by the movement of an object). This speed is equal to the average speed of molecules within the fluid, and it is called the *speed of sound*,[W] noted c.

In fluid dynamics, we often quantify how fast the fluid is flowing relative to the speed of sound. For this, we define the *Mach number*[W] noted [Ma] as the ratio of the local fluid speed V to the local speed of sound c:

$$[\mathrm{Ma}] \equiv \frac{V}{c} \tag{1/10}$$

Since both V and c can be functions of space in a given flow, [Ma] may not be uniform (e.g. the Mach number around an aircraft in flight is different at the nose and above its wings). Nevertheless, a single value is typically chosen to identify "the" representative Mach number of any given flow.

It is observed that providing no heat or work transfer occurs, when fluids flow at $[\mathrm{Ma}] \leq 0{,}3$, their density ρ stays constant. Density variations in practice can be safely neglected below $[\mathrm{Ma}] = 0{,}6$. When the density is uniform, the flow is said to be *incompressible*. Above these Mach numbers, it is observed that when subjected to pressure variations, fluids exert work upon themselves, which translates into measurable density and temperature changes: these are called *compressibility effects*, and we will not study them in this course.

In most flows, the density of *liquids* is almost invariant – so that water flows are generally entirely incompressible.

When the fluid is air (and generally within a perfect gas), it can be shown that c depends only on the absolute temperature:

$$c = \sqrt{\gamma R T} \tag{1/11}$$

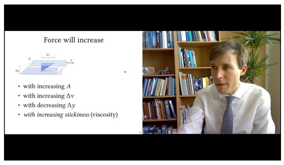
in all cases for a perfect gas,
where c is the local speed of sound ($\mathrm{m\,s^{-1}}$);
γ is a gas property, approx. constant (dimensionless);
and T is the local temperature (K).

Advice from an expert

When you are showering and decrease the temperature at the knob, it takes some time for the water to feel colder: this is because it travels at speed V in the pipe to the showerhead. But when you decrease the *flow rate* at the knob, the response is instantaneous: this is because the decrease in pressure travels at speed c in the pipe ($5\,000\ \mathrm{km\,h^{-1}}$). The dynamics of fluids become funky when the speed of the fluid and the speed of sound are comparable: this is why the Mach number is so important to us.

1.4.6 Viscosity

We have said above that a fluid element can deform continuously under pressure and shear efforts: it will never "snap" or break apart. However this deformation is not "for free": it will require force and energy inputs which are not reversible (they are not reversed if the motion is reversed). Resistance to straining in a fluid is measured with a property named *viscosity*.[w]



Video: how to make sense of viscosity

by Olivier Cleynen (CC-BY)
https://youtu.be/5YUFt-V_kdk

In informal terms, viscosity is the "stickiness" of fluids: for example, honey and sugar syrups are more viscous than water.

More formally, viscosity is quantified as follows. Imagine a small brick-shaped element of fluid, which is deformed (strained) horizontally, as shown in fig. 1.3. The continuous straining of the brick requires a force F per unit area A, called *shear stress* τ (see also §1.5.3 further down). We expect that the shear increases with both the "stickiness" of the fluid —the viscosity— and with the speed Δv at which the brick is strained. Conversely, we expect that the shear will decrease when the element height Δy is increased.

We define the viscosity μ as the ratio between the required shear stress, $\tau = F/A$, and the rate at which the brick is strained, $\Delta v/\Delta y$:

$$\mu \equiv \frac{\tau}{\left(\frac{\Delta v}{\Delta y}\right)} \tag{1/12}$$

where μ is the viscosity (N s m^{-2} or Pa s);
τ is the horizontal shear in fig. 1.3 (Pa);
Δu is the velocity difference between top and bottom planes in fig. 1.3 (m s^{-1});
and Δy is the height difference between top and bottom planes in fig. 1.3 (m).

The dimension of viscosity is in (force per area) divided by (velocity per distance), and so this turns out as Pascal – seconds in SI units. We will come back to shear and viscosity in chapter 5 (*Effects of shear*).

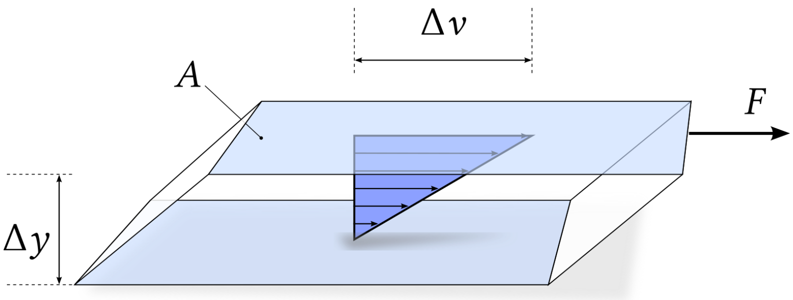


Figure 1.3: A brick-shaped element of fluid is strained, by applying a velocity difference Δv between the top and bottom surface. The horizontal force F required on the top edge, divided by the area A, is the shear stress τ (see also §1.5.3 p. 18 further down). The higher the required shear stress for a given strain rate, the more viscous the fluid is.

Figure CC-0 *Pm.schroeder/Olivier Cleynen*

1.5 Forces on fluids

Fluids are subjected to, and subject their surroundings and themselves to forces. Identifying and quantifying those forces allows us to determine how they will flow. Three types of forces are relevant in fluid dynamics: gravity, pressure and shear.

1.5.1 Gravity

Gravity (here, the attraction effort exerted on fluids at a distance by the Earth) is expressed with a vector $\vec{g}$ pointing downwards. Within the Earth's atmosphere, the magnitude g of this vector varies extremely slightly with altitude, and it may be considered constant at $g = 9{,}81\,\mathrm{m\,s^{-2}}$. The weight force exerted on an object of mass m is then simply quantified as

$$\vec{F}_{\text{weight}} = m\vec{g} \tag{1/13}$$

1.5.2 Pressure

The concept of *pressure*[W] can be approached with the following conceptual experiment: if a flat solid surface is placed in a fluid at zero relative velocity, the pressure p will be the ratio of the perpendicular force $F_\perp$ to the surface area A:

$$p \equiv \frac{F_\perp}{A} \tag{1/14}$$

where p is the pressure ($\mathrm{N\,m^{-2}}$ or Pascals, $1\,\mathrm{Pa} \equiv 1\,\mathrm{N\,m^{-2}}$);
$F_\perp$ is the component of force perpendicular to the surface (N);
and A is the surface area ($\mathrm{m^2}$).

Although in SI units pressure is measured in Pascals ($1\,\mathrm{Pa} \equiv 1\,\mathrm{N\,m^{-2}}$), in practice it is often measured in bars ($1\,\mathrm{bar} \equiv 1 \cdot 10^5\,\mathrm{Pa}$). Ambient atmospheric pressure at normal altitude varies with the weather and is approximately 1 bar.

The pressure distribution within a fluid is related to their velocity distribution according to relations that we will study later. We shall then be looking for a *pressure field* $p_{(x,y,z,t)}$, a function of space an time.

The pressure p exerting on an element of fluid can be thought of as the time- and space-average of the perpendicular component of impact force of its molecules on its neighbors. It is strictly a macroscopic property, that is, it cannot be defined at a microscopic level (there is no such thing as the "pressure of a molecule"). In subsonic flows, it is a scalar property, meaning that for a given particle, it is the same in all directions. Pressure effects are explored in further detail in chapter 4 (*Effects of pressure*).

1.5.3 Shear

In the same thought experiment as above, the *shear*[W], noted τ (greek letter "tau"), expresses the efforts of a force *parallel* to a surface of interest:

$$\tau \equiv \frac{F_\parallel}{A} \tag{1/15}$$

where τ is the shear ($\mathrm{N\,m^{-2}}$ or Pa);
$F_\parallel$ is the component of force parallel to the surface (N);
and A is the surface area ($\mathrm{m^2}$).

Contrary to pressure, shear is not a scalar: it can (and often does) take different values in different directions: on the flat plate above, it would have two components and could be represented by a vector $\vec{\tau} = \left(\tau_x, \tau_y\right)$. We will explore shear in further detail in chapter 5 (*Effects of shear*).

1.6 Basic flow quantities

A few fluid-flow related quantities can be quantified easily and are worth listing here.

Video: how to deal with the ⊥ symbol when calculating mass flow

by Olivier Cleynen (CC-BY)
https://youtu.be/H5l-WlCZQ8

Mass flow[W] is noted $\dot{m}$ and represents the amount of mass flowing through a chosen surface per unit time. When the velocity across the surface is uniform, it can be quantified as:

$$\dot{m} = \rho V_{\perp} A \tag{1/16}$$

where $\dot{m}$ is the mass flow ($\mathrm{kg\,s^{-1}}$);
ρ is the fluid density ($\mathrm{kg\,m^{-3}}$);
A is the area of the considered surface ($\mathrm{m^2}$);
and $V_{\perp}$ is the component of velocity perpendicular to the surface ($\mathrm{m\,s^{-1}}$).

Instead of $V_{\perp}A$, we might pick $VA_{\perp}$: the velocity and the area of a surface *perpendicular* to that velocity:

$$\dot{m} = \rho V A_{\perp} \tag{1/17}$$

where V is the flow speed ($\mathrm{m\,s^{-1}}$);
and $A_{\perp}$ is the area of a surface perpendicular to the flow velocity ($\mathrm{m^2}$).

Volume flow[W] is noted $\dot{\mathcal{V}}$ and represents the volume of the fluid flowing through a chosen surface per unit time. Much like mass flow, when the velocity is uniform, it is quantified as:

$$\dot{\mathcal{V}} = V_{\perp} A = V A_{\perp} = \frac{\dot{m}}{\rho} \tag{1/18}$$

where $\dot{\mathcal{V}}$ is the volume flow ($\mathrm{m^3\,s^{-1}}$).

Power to cross a surface is a time rate of energy transfer. The power $\dot{P}_{\text{pressure}}$ necessary to force a mass flow of fluid through a chosen surface at a pressure p is:

$$\dot{P}_{\text{pressure}} = \vec{F}_{\text{pressure}} \cdot \vec{V}_{\text{fluid}} \tag{1/19}$$
$$= \dot{\mathcal{V}}\, p \tag{1/20}$$
$$= \frac{\dot{m}}{\rho}\, p \tag{1/21}$$

where $\dot{P}_{\text{pressure}}$ is the power required to cross the surface (W);
and p is the mean pressure at the surface (Pa).

If a fluid passes across a *volume*, the net power $\dot{P}_{\text{pressure, net}}$ required to both enter and leave the volume may be expressed as

$$\dot{P}_{\text{pressure, net}} = \frac{\dot{m}}{\rho}\, \Delta p \tag{1/22}$$

where Δp is the pressure difference between outlet and inlet.

Power to increase temperature is also a time rate of energy transfer. If the temperature of a fluid changes when it flows through a volume, as long as no phase change occurs, the associated power $\dot{P}_{\text{temperature}}$ is:

$$\dot{P}_{\text{temperature}} = \dot{m}\, c_{\text{fluid}}\, \Delta T \tag{1/23}$$

where $\dot{P}_{\text{temperature}}$ is the power to increase temperature (W);
ΔT is the temperature change occurring in the fluid (K);
and c_{fluid} is the specific heat capacity of the fluid ($\text{J}\,\text{K}^{-1}\,\text{kg}^{-1}$).

The heat capacity[w] c_{fluid} of fluids varies strongly according to the amount of work that they are performing. When no work is performed in a steady flow, the heat capacity is termed c_p. In fluids such as liquid water and air, this capacity is almost independent of temperature.

As we will see in chapter 2 (*Analysis of existing flows with one dimension*), fluid flow involves many forms of energy changes. We will learn to combine and compare them progressively.

1.7 Four balance equations

Most problems in fluid mechanics are solved by applying basic physical principles. We write out those principles in the form of four *balance equations*.

1. Mass balance:
   The total amount of matter at hand in a given phenomenon must remain constant (since in fluid mechanics, we do not usually consider nuclear reactions). This statement can be expressed as:

$$m_{\text{system}} = \text{cst}$$
$$\frac{\mathrm{d}m_{\text{system}}}{\mathrm{d}t} = 0 \tag{1/24}$$

2. Balance of linear momentum:
   The momentum balance equation is a formulation of Newton's second law,[w] which states that the net force $\vec{F}_{\text{net}} \equiv \Sigma\vec{F}$ applying to any given system is equal to its mass m times its acceleration. In fluid mechanics, the most useful formulation of this physical law uses the change in time of the system's *linear momentum*,[w] the quantity $m\vec{V}$:

$$\vec{F}_{\text{net}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(m\vec{V}\right) \tag{1/25}$$

3. Balance of angular momentum:
   This is a different form of Newton's second law, useful in situations where rotation about an axis, or moments ("twisting" efforts) are applied about an axis. It states that the net moment $\vec{M}_{\text{net, X}} \equiv \Sigma\vec{M}_{\text{X}}$ applied on a system about a point X is equal to the time change of its *angular momentum*[w] about this same point, the quantity $\vec{r} \wedge m\vec{V}$:

$$\vec{M}_{\text{net, X}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{r} \wedge m\vec{V}\right) \tag{1/26}$$

4. Balance of energy:
   This equation, also known as the "first principle of thermodynamics",[w]

states that the total amount of energy within an isolated system must remain constant:

$$\frac{\mathrm{d}E_{\text{isolated system}}}{\mathrm{d}t} = 0 \tag{1/27}$$

In special cases, further equations are used to describe other phenomena affecting the fluid flow (e.g. chemical reactions, or interaction between phases). In most cases however, the four equations above are the only important equations written in fluid mechanics. We usually apply those balance statements to our problem in either one of two ways:

- We may have information about a flow which already exists, and want to calculate how fluid properties change as it flows through the area of interest, and what the related forces are. In that case, we write the equations in an integral form: we will do this in chapters 2 and 3 (*Analysis of existing flows*).
- We may instead wish to predict how the fluid is going to flow through our zone of interest. In order to do this, we need to calculate flow properties in an extensive manner, aiming to obtain vector fields for the velocity and pressure everywhere, at all times. To this effect, we write the equations in a differential form: we will do this in chapter 6 (*Prediction of fluid flows*).

1.8 Classification of fluid flows

As we will see progressively, it is extremely difficult to obtain *general* solutions for fluid flow. Thus, whenever possible or reasonable, simplifying hypothesis are made about the behavior of any particular flow, that allow us to proceed with the analysis and obtain a reasonable, if inexact, specific solution. It is therefore a habit of fluid dynamicists to *classify* fluid flows in various categories, which are not necessarily incompatible. When approaching a given problem, we typically look out for the following characteristics:

Time dependence Flows which do not vary with time are called *steady*. Steadiness is dependent on the chosen point of view: for example, the air flow around an aircraft in cruise flight is seen to be steady from within the aircraft, but is obviously highly unsteady from the point of view of the air particles directly in the path of the airliner. Steadiness is also dependent on the selection of a suitable time frame: time-variations in a flow may be negligible if the time window used in the study is short enough.

Mathematically, flows are steady when either the *partial time derivative* $\partial/\partial t$ or the *total time derivative* $\mathrm{D}/\mathrm{D}t$ —concepts we will study in chapter 6 (*Prediction of fluid flows*) — of properties is zero: this considerably eases the search for solutions. It is important not to confuse steadiness (the fields of properties remain the same as time passes) with *uniformity* (the properties are the same everywhere in space).

To find out whether or not a flow is steady, compare instantaneous representations (photos, measurement readings, vector field representations, etc.) taken at different times. If they are all identical, the flow is steady. There does not exist an analytical method, however, that allows us to *predict* whether a flow will be steady.

Compressibility When the density of the fluid is uniform, the flow is said to be *incompressible*. The term is treacherous, because it refers to density, not pressure (incompressible flows almost always feature non-uniform pressure fields).

To find out whether a gas flow is compressible, compute the Mach number (eq. 1/10 p. 16). Below 0,3 the flow is always incompressible. Compressibility effects can be reasonably neglected below [Ma] = 0,6. Unless very specific phenomena such as phase changes or extreme speeds occur, the flow of liquids is always incompressible.

Advice from an expert

The word “incompressible” is a really mean false friend: it has nothing to do with pressure p, and all with density ρ. Most flows with moderate speeds and powers are incompressible: the density does not change.
Pressure will always change in space or even in time if there is a fluid flow, so there is no word for “pressure remains the same” in fluid mechanics.

Temperature distribution In a fluid flow, temperature changes can occur due to three phenomena:

- Heat transfer from solid bodies;
- Changes in pressure and density due to work being performed on the fluid (by moving solid bodies, or by the fluid itself);
- Heat created through internal friction within the fluid.

To be certain that a fluid flow will have uniform temperature (i.e. whether it is *isothermal*), therefore, we must take three steps:

1. Quantify the heat transfer from external bodies. If that is zero, the flow is at least *adiabatic*, meaning there is no heat transfer;
2. Find out whether the flow is incompressible. As long as it is, the density changes are negligibly small and no temperature changes will occur due to compression or expansion of the fluid;
3. Quantify the mechanical energy lost every second by the fluid as it flows through the domain of interest. This power can be either transmitted to a moving part (e.g. a turbine), or dissipated internally through friction, as heat.
   Because the heat capacity of fluids is generally very high, temperature changes due to internal friction are usually negligibly small. This is assessed with an example in exercise 1.7 p. 29.

Turbulence One last characteristic that we systematically attempt to identify in fluid flows is *turbulence*[W] (or its opposite, *laminarity*). While laminar flows are generally very smooth and steady, turbulent flows feature multiple, chaotic velocity field variations in time and space.

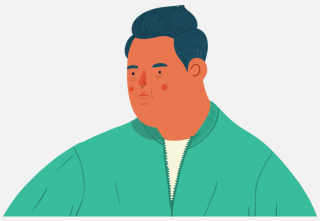
We shall first approach the concept of turbulence in chapter 7 (*Pipe flows*), and study it more formally in chapter 9 (*Dealing with turbulence*). In the meantime, we can predict whether a flow will become turbulent by using a non-dimensional parameter named the *Reynolds number*, noted [Re]:

$$[\mathrm{Re}] \equiv \frac{\rho V L}{\mu} \tag{1/28}$$

To find out whether a flow will become turbulent, quantify [Re] by using the fluid properties ρ and μ, a representative fluid speed V, and a length L which is representative of the flow domain (for example, the length or width of an obstacle in the flow). If the result is on the order of 10^4 or more, the flow is very likely to become turbulent over the length L. By contrast, with [Re] of the order of 10^2 or less, the flow is very likely to remain laminar over this length.

This crude quantification, of course, deserves more explanation — we will be coming back to the Reynolds number in chapters 7 and following.

Advice from an expert



The Reynolds number is the best measure of how complex a flow is — how intricate and chaotic the movement of the fluid will be. When they want to compare themselves to their peers (and maybe try to impress them), fluid dynamicists won't ask about flow speed or mass flow, but instead just ask: "what's your Reynolds number?"

1.9 Limits of fluid dynamics

Fluid dynamics is a complex discipline.

It is easy to observe that flows as ordinary as sea waves crashing on a reef, water flowing down a river stream, or air blown into one's hands, display tremendous geometrical complexity. Even after choosing to describe only the movement of macroscopic fluid particles instead of individual molecules (and thereby avoiding studying thousands of billions of individual movements), we still need to describe a three-dimensional field of properties (pressure, temperature, etc.), one of which, velocity, is itself three-dimensional.

Thus, even before we begin describing the exact problem and a procedure to obtain its solution, we know that the mere description of a *solution* can have tremendous complexity.

Additionally, we have to admit that in practice much progress can be made in the field of fluid dynamics. For example, our weather forecasts have almost no value beyond one week, and aircraft manufacturers with budgets measured in billions of dollars still make extensive use of wind tunnel models – this despite our staggering continuous rate of progress in computing technology, and many decades of efforts dedicated to analytical fluid dynamics.

In our present study of fluid dynamics, therefore, we shall proceed modestly, and will always take care to show the limits of our analysis.

Force due to pressure on a plate

A large, 3×3 m square aquarium window (fig. 1.4) is subjected to pressure from fluids on each side. On the right side, the atmosphere exerts uniform pressure p_{air} as:

$$p_{\text{air}} = p_{\text{atmosphere}} \tag{1/29}$$

On the other side, water from the aquarium exerts non-uniform pressure p_{water} expressed in Pascals as:

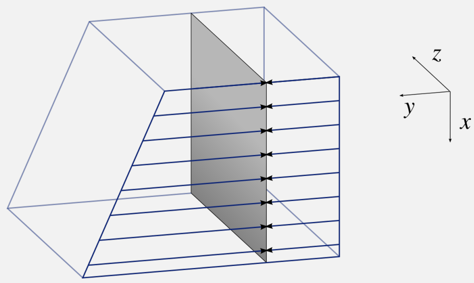
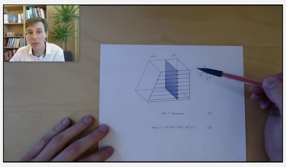


Figure 1.4: Pressure distribution on a plate: on the left, exerted by water; on the right, exerted by air.

Figure CC-0 *Olivier Cleynen*

$$p_{\text{water}} = 1{,}2 \cdot 10^5 + 9{,}81 \cdot 10^3 \times x \tag{1/30}$$

What is the force resulting from fluid pressure on each side of the plate?



See this solution worked out step by step on YouTube
https://youtu.be/2PE74f6fIMM (CC-BY Olivier Cleynen)

Note: Unfortunately Olivier made an error in this video: the end computation is incorrect! The correct result is 1,2124 MN. The method, numbers, equations etc. are all ok — only the final result is affected. Many thanks to the students who double-checked and reported the problem!

Power and moment resulting from a force

A sailboat travels at velocity $\vec{V}$ with V=1,5 m s^{-1} (see fig. 1.5). The relative wind comes from the back and from the left; it acts on the sail and results in an aerodynamic force $\vec{F}$. The magnitude is F=13 kN; the force acts at an angle θ=30°, 2 m ahead of the center of gravity.

What is the power contributed by the wind, and what is the moment exerted by the aerodynamic force about the center of gravity?

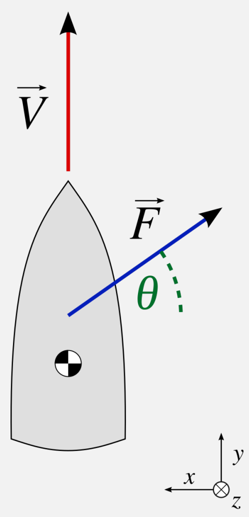
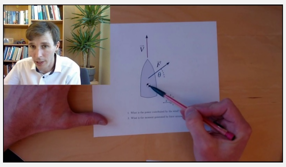


Figure 1.5: A boat seen from above, traveling at velocity $\vec{V}$. The wind exerts a force $\vec{F}$ on the boat, a little forward of its center of gravity.



See this solution worked out step by step on YouTube

https://youtu.be/yLqsGF-R9ps

Problem sheet 1: Basic flow quantities

last edited September 3, 2020
by Olivier Cleynen — https://fluidmech.ninja/

Except otherwise indicated, assume that:
The atmosphere has $p_{\text{atm.}} = 1\,\text{bar}$; $\rho_{\text{atm.}} = 1{,}225\,\text{kg}\,\text{m}^{-3}$; $T_{\text{atm.}} = 11{,}3\,°\text{C}$; $\mu_{\text{atm.}} = 1{,}5 \cdot 10^{-5}\,\text{Pa}\,\text{s}$
Air behaves as a perfect gas: R_{air}=287 J kg^{-1} K^{-1}; γ_{air}=1,4; $c_{p\ \text{air}}$=1 005 J kg^{-1} K^{-1}; $c_{v\ \text{air}}$=718 J kg^{-1} K^{-1}
Liquid water is incompressible: $\rho_{\text{water}} = 1\,000\,\text{kg}\,\text{m}^{-3}$, $c_{p\ \text{water}} = 4\,180\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

1.1 Reading quiz

Once you are done with reading the content of this chapter, you can go take the associated quiz at https://elearning.ovgu.de/course/view.php?id=7199

In the winter semester, quizzes are not graded.

1.2 Compressibility effects

An aircraft is flying in air with density 0,9 kg m^{-3} and temperature −5 °C. Above which flight speed would you expect the air flow over the wings to become compressible?

1.3 Pressure-induced force

A flat, 2 m-by-2 m panel is used as the wall of a swimming pool (fig. 1.6). On the left side, the pressure is uniform at 1 bar.

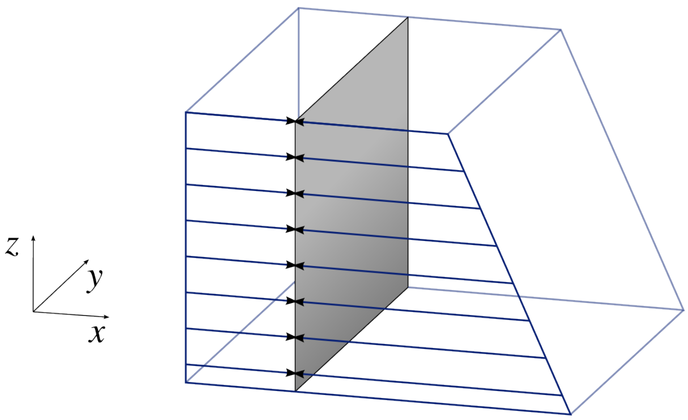


Figure 1.6: Pressure distribution on a flat panel that is part of the wall of a swimming pool

Figure CC-0 *Olivier Cleynen*

1.3.1. What is the pressure force (i.e. the force resulting from the pressure) exerted on the left side of the plate?

On the right side of the plate, the water exerts a pressure which is not uniform: it increases with depth. The relation, expressed in pascals, is:

$$p_{\text{water}} = 1{,}3 \cdot 10^5 - 9{,}81 \cdot 10^3 \times z \qquad (1/31)$$

1.3.2. What is the pressure force exerted on the right side of the plate?
[Hint: we will explore the required expression in chapter 4 (Effects of pressure) as eq. 4/3 p. 74]

1.4 Shear-induced force

A fluid flows over a 3 m by 3 m flat horizontal plate, in the x-direction as shown in fig. 1.7. Because of this flow, the plate is subjected to uniform shear $\tau_{zx} = 1{,}65\,\mathrm{Pa}$.

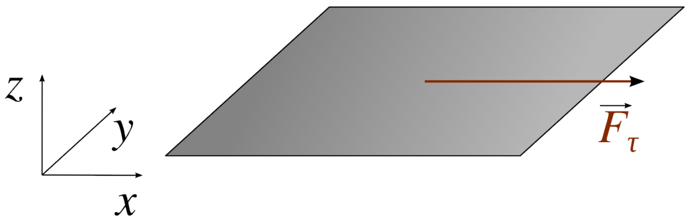


Figure 1.7: Shear force exerting on a plate

Figure CC-0 *Olivier Cleynen*

1.4.1. What is the shear force applying on the plate?

1.4.2. What would be the shear force if the shear was not uniform, but instead was a function of x expressed (in pascals) as $\tau_{zx} = 1{,}65 - 0{,}01 \times x^2$?
[Hint: we will explore the required expression in chapter 5 (Effects of shear) as eq. 5/3 p. 92]

1.5 Speed of sound

White [22] P1.87

Isaac Newton measured the speed of sound by timing the interval between observing smoke produced by a cannon blast and the hearing of the detonation.

The cannon is shot 8,4 km away from Newton. What is the air temperature if the measured interval is 24,2 s? What is the temperature if the interval is 25,1 s?

1.6 Wind on a truck

A truck moves with constant speed $\vec{V}$ on a road, with $V = 50\,\mathrm{km\,h^{-1}}$. It experiences strong relative wind coming from the back and from the right: this results in an aerodynamic force $\vec{F}$ with $F = 5\,\mathrm{kN}$ at an angle $\theta = 20°$, as shown in fig. 1.8.

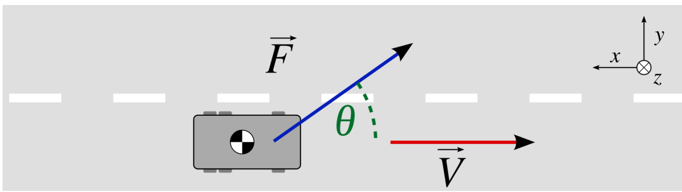


Figure 1.8: Top view of a truck traveling at velocity $\vec{V}$ and subject to a aerodynamic force $\vec{F}$

Figure CC-0 *Olivier Cleynen*

1.6.1. What is the power given by the wind to the truck?

The force $\vec{F}$ is applying at a distance 0,8 m behind the center of gravity of the truck.

1.6.2. What are the magnitude and the direction of the moment exerted by the aerodynamic force $\vec{F}$ about the center of gravity?

1.7 Go-faster exhaust pipe

The engine exhaust gases of a student's hot-rod car are flowing quasi-steadily in a cylindrical outlet pipe, whose outlet is slanted at an angle $\theta = 25°$ to improve the good looks of the car and provide the opportunity for an exercise.

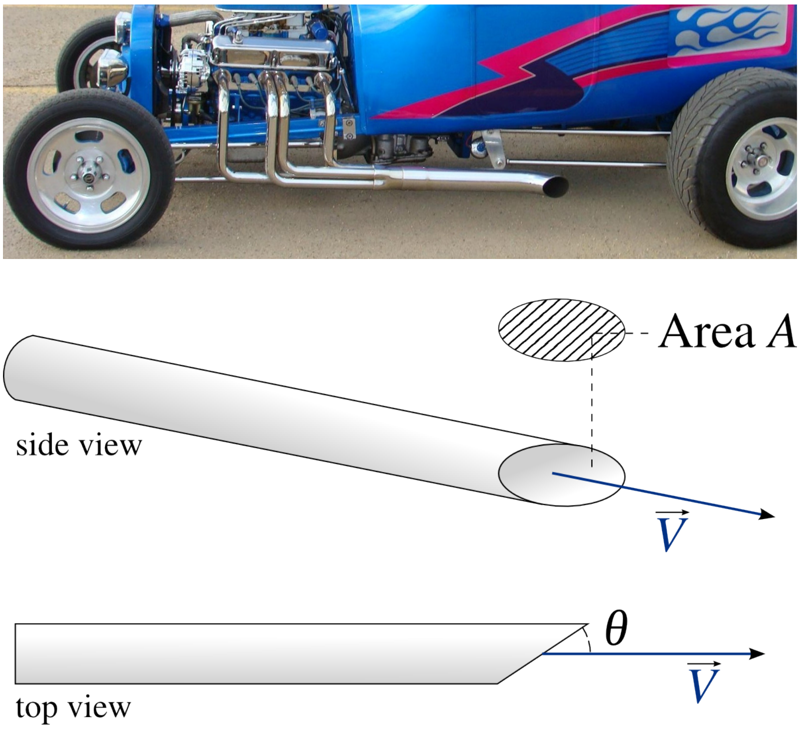


Figure 1.9: Exhaust gas pipe of a car. The outlet cross-section is at an angle θ relative to the axis of the pipe.

The outlet velocity is measured at 15 m s^{-1}, and the exhaust gas density is 1,1 kg m^{-3}. The slanted outlet section area A is 420 cm^2.

1.7.1. What is the mass flow $\dot{m}$ through the pipe?

1.7.2. What is the volume flow $\dot{V}$ of exhaust gases?

Because of the shear within the exhaust gases, the flow through the pipe induces a pressure loss of 21 Pa – we will learn to quantify this in chapter 7 (*Pipe flows*). In these conditions, the specific heat capacity of the exhaust gases is $c_{p\text{gases}} = 1\,100$ J kg^{-1} K^{-1}.

1.7.3. What is the power required to carry the exhaust gases through the pipe?

1.7.4. What is the gas temperature increase due to the shear in the flow?

1.8 Acceleration of a particle

Inside a complex, turbulent water flow, we are studying the trajectory of a cubic fluid particle of width 0,1 mm. The particle is accelerating at a rate of $2{,}5\,\mathrm{m\,s^{-2}}$.

1.8.1. What is the net force applying to the particle?

1.8.2. In practice, which types of forces could cause it to accelerate?

1.9 Flow classifications

1.9.1. Can an incompressible flow also be unsteady?

1.9.2. Can a very viscous fluid flow in a turbulent manner?

1.9.3. *[more difficult]* Can a compressible flow also be isothermal?

1.9.4. Give an example of an isothermal flow, of an unsteady flow, of a compressible flow, and of an incompressible flow.

Answers

1.2 If you adopt [Ma] = 0,6 as an upper limit, you will obtain $V_{\text{max}} = 709\,\text{km}\,\text{h}^{-1}$ (eqs. 1/10 & 1/11 p. 16). Note that propellers, fan blades etc. will meet compressiblity effects far sooner.

1.3 1) $F_{\text{left}} = 400\,\text{kN}$ (eq. 1/14 p. 18); 2) $F_{\text{right}} = 480\,\text{kN}$ (eq. 4/3 p. 74).

1.4 1) $F_1 = 14{,}85\,\text{N}$ (eq. 1/15 p. 18); 2) $F_2 = 14{,}58\,\text{N}$ (eq. 5/3 p. 92).

1.5 26,7 °C & 5,6 °C.

1.6 1) $\dot{W} = \vec{F}_{\text{aero}} \cdot \vec{V}_{\text{truck}} = 65{,}3\,\text{kW}$. See Appendix A2.1 p. 247 for a short briefing about the dot product of vectors;
2) $M = ||\vec{r} \wedge \vec{F}_{\text{aero}}|| = 1\,368\,\text{N}\,\text{m}$, $\vec{M} = \begin{pmatrix} 0 \\ 0 \\ -1\,368 \end{pmatrix}$ (points vertically upwards). See Appendix A2.2 p. 248 for a short briefing about the cross product of vectors.

1.7 1) $\dot{m} = 0{,}2929\,\text{kg}\,\text{s}^{-1}$ (eq. 1/16 p. 19);
2) $\dot{\mathcal{V}} = 266{,}2\,\text{L}\,\text{s}^{-1}$ (eq. 1/18 p. 19);
3) $\dot{W} = 5{,}59\,\text{W}$ (eq. 1/20 p. 19);
4) $\Delta T = +0{,}0174\,\text{K}$ (eq. 1/23 p. 20), an illustration of remarks made in §1.8 p. 22 regarding temperature distribution.

1.8 1) $F_{\text{net}} = 2{,}5 \cdot 10^{-9}\,\text{N}$ (eq 1/25 p. 20), such are the orders of magnitude involved in CFD calculations!
2) Only three kinds: forces due to pressure, shear, and gravity.

1.9 1) yes, 2) yes if [Re] is high enough, 3) yes (in very specific cases such as high pressure changes combined with high heat transfer or high irreversibility, therefore generally no), 4) open the cap of a water bottle and turn it upside down: you have an isothermal, unsteady, incompressible flow. An example of compressible flow could be the expansion in a jet engine nozzle.