Fluid Mechanics
Chapter 0 – Important concepts

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These lecture notes are based on textbooks by White [12], Çengel & al.[15], and Munson & al.[17].

0.1 Concept of a fluid

We call *fluid* a type of matter which is continuously deformable, and which spontaneously tends to adapt its shape to its container by occupying all of the space made available to it.

0.2 Fluid mechanics

0.2.1 Solution of a flow

Fluid mechanics is the study of fluids subjected to given constraints. The most common type of problem in this discipline is the search for a complete description of the fluid flow around or through a solid object. This problem is solved when the entire set of velocities of fluid particles has been described. This set of velocities, which is often a function of time, can be described either as a set of discrete values (“pixel” data) or as a mathematical function; it is called the *flow solution*.

Once this solution is known, the shear and pressure efforts generated on the surface of the object can be calculated. Other quantities, such as force, moment, energy gain or losses, can then be deduced.

0.2.2 Modeling of fluids

Like all matter, fluids are made of discrete, solid molecules. However, within the scope of fluid mechanics we work at the macroscopic scale, in which matter can be treated like a *continuum* within which all physical properties of interest can be continuously differentiable.

There are about $2 \cdot 10^{22}$ molecules in the air within an "empty" 1-liter bottle at ambient temperature and pressure. Even when the air within the bottle is completely still, these molecules are constantly colliding with each other and with the bottle walls; on average, their speed is equal to the speed of sound: approximately 1 000 km/h.

Despite the complexity of individual molecule movements, even the most turbulent flows can be accurately described and solved by considering the velocities of *groups* of several millions of molecules collectively, which we name *fluid particles*. By doing so, we never find out the velocity of individual molecules: instead, those are averaged in space and time and result in much simpler and smoother trajectories, which are those we can observe with macroscopic instruments such as video cameras and pressure probes.

Our selection of an appropriate fluid particle size (in effect defining the lower boundary of the *macroscopic scale*), is illustrated in fig. 0.1. We choose to reduce our volume of study to the smallest possible size before the effect of individual molecules becomes meaningful.

Adopting this point of view, which is named the *continuum abstraction*, is not a trivial decision, because the physical laws which determine the behavior of molecules are very different from those which determine the behavior of elements of fluid. For example, in fluid mechanics we never consider any inter-element attraction or repulsion forces; while new forces "appear" due to pressure or shear effects that do not exist at a molecular level.

A direct benefit of the continuum abstraction is that the mathematical complexity of our problems is greatly simplified. Finding the solution for the bottle of "still air" mentioned above, for example, requires only a single

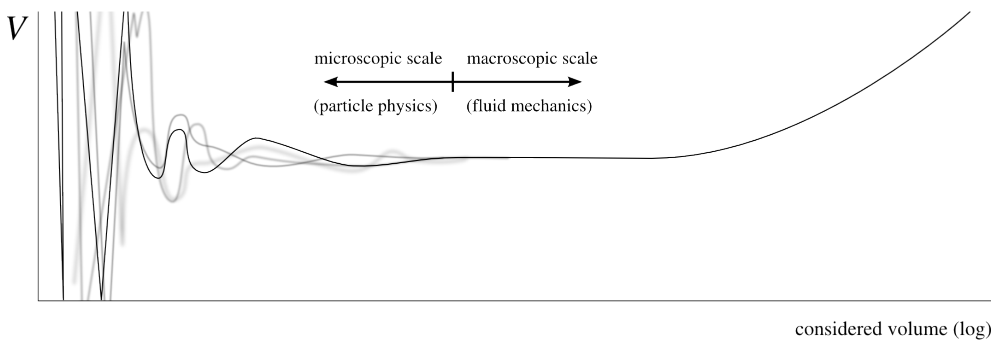

Figure 0.1 – Measurement of the average value of a property (here, velocity V; but it could be pressure, or temperature) inside a given volume. As the volume shrinks towards zero, the fluid can no longer be treated as a continuum; and property measurements will oscillate wildly.

Figure CC-BY-SA O.C.

equation (section 1.3.3 p.33) instead of a system of $2 \cdot 10^{22}$ equations with $2 \cdot 10^{22}$ unknowns (all leading up to $\vec{V}_{\text{average},x,y,z,t} = \vec{0}$!).

Another consequence is that we cannot treat a fluid as if it were a mere set of marbles with no interaction which would hit objects as they move by. Instead we must think of a fluid –even a low-density fluid such as atmospheric air– as an infinitely-flexible medium able to fill in almost instantly all of the space made available to it.

0.2.3 Theory, numerics, and experiment

Today, fluid dynamicists are typically specializing in any one of three sub-disciplines:

Analytical fluid mechanics which is the main focus of these lectures and which consists in predicting fluid flows mathematically. As we shall see, it is only able to provide (exact) solutions for very simple flows. In fluid mechanics, theory nevertheless allows us to understand the mechanisms of complex fluid phenomena, describe scale effects, and predict forces associated with given fluid flows;

Numerical fluid mechanics also called *Computational Fluid Dynamics* or CFD, which consists in solving problems using very large amounts of discrete values. Initiated as a research topic in the 1970s, CFD is now omnipresent in the industry; it allows for excellent visualization and parametric studies of very complex flows. Nevertheless, computational solutions obtained within practical time frames are inherently approximate: they need to be challenged using analysis, and calibrated using experimental measurements;

Experimental fluid mechanics which consists in reproducing phenomena of interest within laboratory conditions and observing them using experimental techniques. A very mature branch (it first provided useful results at the end of the 19th century), it is unfortunately associated with high human, equipment and financial costs. Experimental measurements are indispensable for the validation of computational simulations; meanwhile, the design of meaningful experiments necessitates a good understanding of scale effects.

Our study of analytical fluid mechanics should therefore be a useful tool to approach the other two sub-disciplines of fluid mechanics.

0.3 Important concepts in mechanics

Mechanics in general deals with the study of forces and motion of bodies. A few concepts relevant for us are recalled here.

0.3.1 Position, velocity, acceleration

The description of the movement of bodies (without reference to the causes and effects of that movement) is called *kinematics*.

The *position* in space of an object can be fully expressed using three components (one for each dimension) This is usually done with a vector (here written $\vec{x}$). If the object moves, then this vector varies with time.

The *velocity* $\vec{V}$ of the object is the rate of change in time of its position:

$$\vec{V} \equiv \frac{d\vec{x}}{dt} \tag{0/1}$$

The magnitude of velocity is called *speed* and measured in $m\,s^{-1}$. Contrary to speed, in order to express velocity completely, three distinct values (also each in $m\,s^{-1}$) must be expressed. Generally, this is done by using one of the following notations:

$$\begin{aligned}\vec{V} &= \begin{pmatrix} u \\ v \\ w \end{pmatrix} \\ &= \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \\ &= (u_i) \equiv (u_1, u_2, u_3) \\ &= \begin{pmatrix} u_r \\ u_\theta \\ u_z \end{pmatrix}\end{aligned}$$

The *acceleration* $\vec{a}$ of the object is the rate of change in time of its velocity:

$$\vec{a} \equiv \frac{d\vec{V}}{dt} \tag{0/2}$$

Acceleration is especially important in mechanics because it can be deduced from Newton's second law (see eq. 0/23 p.19 below) if the forces applying on the object are known. Acceleration can then be integrated with respect to time to obtain velocity, which can be integrated with respect to time to obtain position.
Like velocity, acceleration has three components (each measured in $m\,s^{-2}$). It shows at which rate each component of velocity is changing. It may not always point in the same direction as velocity (fig. 0.2).

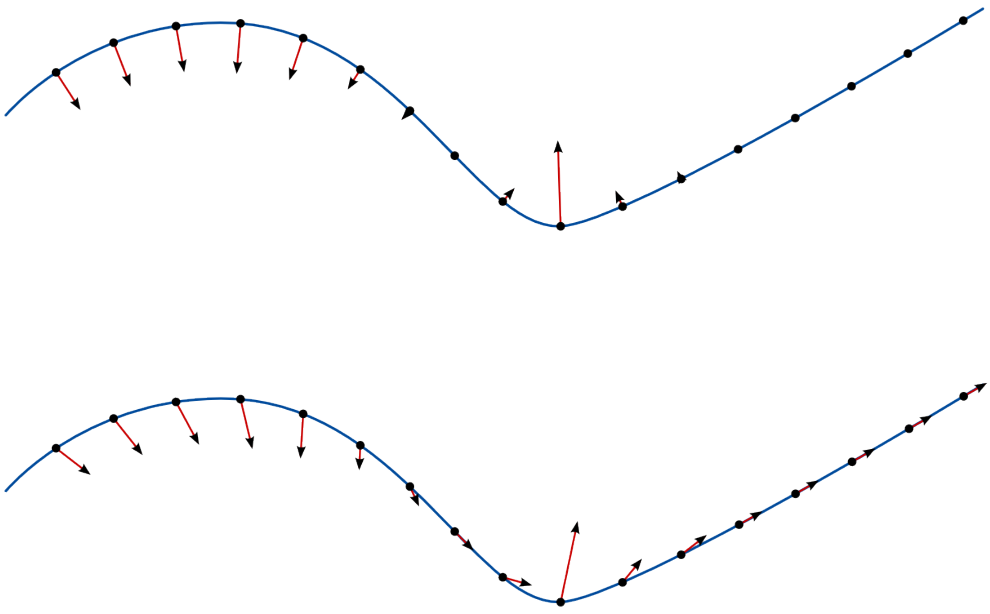

Figure 0.2 – Acceleration vectors of a body following a curved trajectory when its speed (the magnitude of its velocity) remains constant (top) and when it changes continuously (bottom)

Figure CC-0 o.c.

0.3.2 Forces and moments

A *force* expresses an effort exerted on a body. Force has a direction as well as a magnitude, and so it is expressed with a vector (typically noted $\vec{F}$). The three main types of forces relevant to fluid mechanics are those due to pressure, those due to shear, and those due to gravity (the force due to gravity is called *weight*).

When a force exerts at a position $\vec{r}$ away from a reference point, it exerts a torsion (or "twisting") effort named *moment*. Like a force, a moment has a direction as well as a magnitude, and so is best expressed with a vector. This vector $\vec{M}$ is expressed as the cross-product of the *arm* $\vec{r}$ and the force $\vec{F}$:

$$\vec{M} \equiv \vec{r} \wedge \vec{F} \qquad (0/3)$$

0.3.3 Energy

Energy, measured in joules (J), is in most general terms the ability of a body to set other bodies in motion. It can be accumulated or spent by bodies in a large number of different ways. The most relevant forms of energy in fluid mechanics are:

Kinetic energy E_k, accumulated as motion

$$E_\text{k} \equiv \frac{1}{2} m V^2 \qquad (0/4)$$

Work W, which is energy spent on displacing an object over a distance l with a force F:

$$W \equiv \vec{F} \cdot \vec{l} \qquad (0/5)$$

Internal energy I stored as heat within the body itself. As long as no phase changes occurs, the internal energy I of fluids is roughly proportional to their absolute temperature T.

0.4 Properties of fluids

Beyond velocity, which is the primary unknown for us in fluid mechanics, there are a few other important fluid properties.

0.4.1 Density

The density ρ is defined as the inverse of mass-specific volume v,

$$\rho \equiv \frac{1}{v} = \frac{m}{\mathcal{V}} \tag{0/6}$$

where ρ the density ($\text{kg}\,\text{m}^{-3}$);
v is the specific volume ($\text{m}^3\,\text{kg}^{-1}$);
m is the considered mass (kg);
and $\mathcal{V}$ is the considered volume (m^3).

At ambient conditions air has a density of approximately $\rho_{\text{air}} = 1{,}2\,\text{kg}\,\text{m}^{-3}$; that of water is almost a thousand times greater, at $\rho_{\text{water}} = 1\,000\,\text{kg}\,\text{m}^{-3}$.

0.4.2 Phase

Fluids can be broadly classified into *phases*, which are loosely-defined sets of physical behaviors. Typically one distinguishes *liquids* which are fluids with large densities on which surface tension effects play an important role, from *gases* or *vapors* which have low densities and no surface tension effects. Phase changes are often brutal (but under specific conditions can be blurred or smeared-out); they usually involve large energy transfers. The presence of multiple phases in a flow is an added layer of complexity in the description of fluid phenomena.

0.4.3 Temperature

Temperature is a scalar property measured in Kelvins (an absolute scale). It represents a body's potential for receiving or providing heat and is defined, in thermodynamics, based on the transformation of heat and work.

A conversion between Kelvins and degrees Celsius by subtracting 273,15 units:

$$T(^\circ\text{C}) = T(\text{K}) - 273{,}15 \tag{0/7}$$

Although we can "feel" temperature in daily life, it must be noted that the human body is a very poor thermometer in practice. This is because we constantly produce heat, and we infer temperature by the power our body loses or gains as heat. This power not only depends on our own body temperature (hot water "feels" hotter when we are cold), but also on the heat capacity of the fluid (cold water "feels" colder than air at the same temperature) and on the amount of convection taking place (ambient air "feels" colder on a windy day).

In spite of these impressions, the fact is that the heat capacity of fluids is extremely high ($c_{\text{air}} \approx 1\,\text{kJ}\,\text{kg}^{-1}\,\text{K}$, $c_{\text{water}} \approx 4\,\text{kJ}\,\text{kg}^{-1}\,\text{K}$). Unless very high velocities are attained, the temperature changes associated with fluid flow are much too small to be measurable in practice.

0.4.4 Perfect gas model

Under a specific set of conditions (most particularly at high temperature and low pressure), several properties of a gas can be related easily to one another. The absolute temperature T is then modeled as a function of their pressure p with a single, approximately constant parameter $R \equiv pv/T$:

$$pv = RT \tag{0/8}$$

$$\frac{p}{\rho} = RT \tag{0/9}$$

where R depends on the state and nature of the gas ($\text{J}\,\text{K}^{-1}\,\text{kg}^{-1}$);
and p is the pressure (Pa), defined later on.

This type of model (relating temperature to pressure and density) is called an *equation of state*. Where R remains constant, the fluid is said to behave as a *perfect gas*. The properties of air can satisfactorily be predicted using this model. Other models exist which predict the properties of gases over larger property ranges, at the cost of increased mathematical complexity.

Many fluids, especially liquids, do not follow this equation and their temperature must be determined in another way, most often with the help of laboratory measurements.

0.4.5 Speed of sound

An important property of fluids is the speed at which information (in particular, pressure changes due to the movement of an object) can travel within the fluid. This is equal to the average speed of molecules within the fluid, and it is called the *speed of sound*, noted a.

It is important to quantify how fast the fluid is flowing relative to e speed of sound. For this, we define the *Mach number* [Ma] as the ratio of the local fluid speed V to the local speed of sound a:

$$[\text{Ma}] \equiv \frac{V}{a} \tag{0/10}$$

Since both V and a can be functions of space in a given flow, [Ma] may not be uniform (e.g. the Mach number around an airliner in flight is different at the nose than above the wings). Nevertheless, a representative Mach number can typically be found for any given flow.

It is observed that providing no heat or work transfer occurs, when fluids flow at $[\text{Ma}] \leq 0{,}3$, their density ρ stays constant. Density variations in practice can be safely neglected below $[\text{Ma}] = 0{,}6$. These flows are termed *incompressible*. Above these Mach numbers, it is observed that when subjected to pressure variations, fluids exert work upon themselves, which translates into measurable density and temperature changes: these are called *compressibility effects*, and we will study them in chapter 9.

In most flows, the density of *liquids* is almost invariant – so that water flows are generally entirely incompressible.

When the fluid is air (and generally within a perfect gas), we will show in chapter 9 (§9.3 p.196) that a depends only on the absolute temperature:

$$a = \sqrt{\gamma R T} \tag{0/11}$$

in all cases for a perfect gas,
where a is the local speed of sound ($\mathrm{m\,s^{-1}}$);
γ is a gas property, approx. constant (dimensionless);
and T is the local temperature (K).

0.4.6 Viscosity

We have accepted that a fluid element can deform continuously under pressure and shear efforts; however this deformation may not always be "for free", i.e. it may require force and energy inputs which are not reversible. Resistance to strain in a fluid is measured with a property named *viscosity*.

In informal terms, viscosity is the "stickiness" of fluids, i.e. their resistance to being sheared: honey and sugar syrups are very viscous while water has low viscosity.

Formally, viscosity is defined as the ratio between the time rate of deformation of a fluid (its strain rate), and the shear effort applied to it. For example, if a fluid element is strained uniformly in the x-direction, the velocity u_x will be different everywhere inside the fluid element, and shear τ_{yx} (the shear in the x-direction in the plane perpendicular to the y-direction) will occur. The viscosity μ is then defined as:

$$\mu = \frac{\tau_{xy}}{\left(\frac{\partial u_x}{\partial y}\right)} \tag{0/12}$$

where μ is the viscosity ($\mathrm{N\,s\,m^{-2}}$ or $\mathrm{Pa\,s}$);
τ_{xy} is the shear in the x-direction along the plane perpendicular to the y-direction (Pa);
and u_x is the velocity in the x-direction ($\mathrm{m\,s^{-1}}$).

This equation 0/12 can be generalized for all directions (i, j and k) as:

$$\mu \equiv \frac{\tau_{ij}}{\left(\frac{\partial u_i}{\partial j}\right)} \tag{0/13}$$

For most fluids, viscosity is the same in all directions and for all strain rates. This characteristic makes them *Newtonian fluids*. Viscosity is affected by temperature. We will study the effects of shear and viscosity in chapter 2.

0.5 Forces on fluids

Fluids are subjected to, and subject their surroundings and themselves to forces. Identifying and quantifying those forces allows us to determine how they will flow. Three types of forces are relevant in fluid dynamics: gravity, pressure and shear.

0.5.1 Gravity

Gravity (here, the attraction effort exerted on fluids at a distance by the Earth) is expressed with a vector $\vec{g}$ pointing downwards. Within the Earth's atmosphere, the magnitude g of this vector varies extremely slightly with altitude, and it may be considered constant at $g = 9{,}81\,\mathrm{m\,s^{-2}}$. The weight force exerted on an object of mass m is then simply quantified as

$$\vec{F}_{\text{weight}} = m\vec{g} \tag{0/14}$$

0.5.2 Pressure

The concept of pressure can be approached with the following conceptual experiment: if a flat solid surface is placed in a fluid at zero relative velocity, the pressure p will be the ratio of the perpendicular force $F_\perp$ to the surface area A:

$$p \equiv \frac{F_\perp}{A} \tag{0/15}$$

where p is the pressure ($\mathrm{N\,m^{-2}}$ or pascals, $1\,\mathrm{Pa} \equiv 1\,\mathrm{N\,m^{-2}}$);
$F_\perp$ is the component of force perpendicular to the surface (N);
and A is the surface area ($\mathrm{m^2}$).

Although in SI units pressure is measured in Pascals ($1\,\mathrm{Pa} \equiv 1\,\mathrm{N\,m^{-2}}$), in practice it is often measured in bars ($1\,\mathrm{bar} \equiv 1 \cdot 10^5\,\mathrm{Pa}$). Ambient atmospheric pressure at normal altitude varies with the weather and is approximately 1 bar.

The pressure distribution within a fluid is related to their velocity distribution according to relations that we will study later. We shall then be looking for a *pressure field* $p_{(x,y,z,t)}$, a function of space an time.

The pressure p exerting on an element of fluid can be thought of as the time- and space-average of the perpendicular component of impact force of its molecules on its neighbors. It is strictly a macroscopic property, that is, it cannot be defined at a microscopic level (there is no such thing as the "pressure of a molecule"). In subsonic flows, it is a scalar property, meaning that for a given particle, it is the same in all directions. Pressure effects are explored in further detail in chapter 1.

0.5.3 Shear

In the same thought experiment as above, shear τ expresses the efforts of a force *parallel* to a surface of interest:

$$\tau \equiv \frac{F_\parallel}{A} \tag{0/16}$$

where τ is the shear ($\mathrm{N\,m^{-2}}$ or Pa);
$F_\parallel$ is the component of force parallel to the surface (N);

and A is the surface area (m^2).

Contrary to pressure, shear is not a scalar, i.e. it can (and often does) take different values in different directions: on the flat plate above, it would have two components and could be represented by a vector $\vec{\tau} = \left(\tau_x, \tau_y\right)$. We will explore shear in further detail in chapter 2.

0.6 Basic flow quantities

A few fluid-flow related quantities can be quantified easily and are worth listing here.

Mass flow is noted $\dot{m}$ and represents the amount of mass flowing through a chosen surface per unit time. When the velocity across the surface is uniform, it can be quantified as:

$$\dot{m} = \rho V_{\perp} A \tag{0/17}$$

where $\dot{m}$ is the mass flow ($kg\,s^{-1}$);
ρ is the fluid density ($kg\,s^{-1}$);
A is the area of the considered surface (m^2);
and $V_{\perp}$ is the component of velocity perpendicular to the surface ($m\,s^{-1}$).

Alternatively, one might consider instead the full fluid velocity and the area of a surface *perpendicular* to that velocity:

$$\dot{m} = \rho V A_{\perp} \tag{0/18}$$

where V is the flow speed ($m\,s^{-1}$);
and $A_{\perp}$ is the area of a surface perpendicular to the flow velocity (m^2).

Volume flow is noted $\dot{\mathcal{V}}$ and represents the volume of the fluid flowing through a chosen surface per unit time. Much like mass flow, when the velocity is uniform, it is quantified as:

$$\dot{\mathcal{V}} = V_{\perp} A = V A_{\perp} = \frac{\dot{m}}{\rho} \tag{0/19}$$

where $\dot{\mathcal{V}}$ is the volume flow ($m^3\,s^{-1}$).

Mechanical power is a time rate of energy transfer. Compression and expansion of fluids involves significant power. When the flow is incompressible (see §0.8 below), the mechanical power $\dot{W}$ necessary to force a fluid through a fixed volume where pressure losses occur is:

$$\dot{W} = \vec{F}_{\text{net, pressure}} \cdot \vec{V}_{\text{fluid, average}} = \dot{\mathcal{V}}\ |\Delta p|_{\text{loss}} \tag{0/20}$$

where $\dot{W}$ is the power spent as work (W);
and $|\Delta p|$ is the pressure loss occurring due to fluid flow (Pa).

Power as heat is also a time rate of energy transfer. When fluid flows uniformly and steadily through a fixed volume, its temperature T increases according to the net power as heat $\dot{Q}$:

$$\dot{Q} = \dot{m}\ c_{\text{fluid}} \Delta T \tag{0/21}$$

where $\dot{Q}$ is the power spent as heat (W);
ΔT is the temperature change occurring in the fluid (K);
and c_{fluid} is the specific heat capacity of the fluid ($J\,K^{-1}\,kg^{-1}$).

The heat capacity of fluids varies strongly according to the amount of work that they are performing. When no work is performed in a steady flow, the heat capacity is termed c_p. In fluids such as liquid water and air, this capacity is almost independent from temperature.

0.7 Five equations

The analytical and numerical resolution of problems in fluid mechanics is arched upon five important physical principles, which are sometimes called *conservation principles*.

1. Mass conservation:
   Without nuclear reactions, the total amount of matter at hand in a given phenomenon must remain constant. This statement can be expressed as:

$$m_{\text{system}} = \text{cst}$$
$$\frac{\mathrm{d}m_{\text{system}}}{\mathrm{d}t} = 0 \tag{0/22}$$

2. Conservation of linear momentum:
   This is a form of Newton's second law, which states that the net force $\vec{F}_{\text{net}} \equiv \Sigma\vec{F}$ applying to any given system of mass m is equal to the time change of its linear momentum $m\vec{V}$:

$$\vec{F}_{\text{net}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(m\vec{V}\right) \tag{0/23}$$

3. Conservation of angular momentum:
   This is a different form of Newton's second law, which states that the net moment $\vec{M}_{\text{net, X}} \equiv \Sigma\vec{M}_{\text{X}}$ applied on a system about a point X is equal to the time change of its total angular momentum about this same point $\vec{r} \wedge m\vec{V}$:

$$\vec{M}_{\text{net, X}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{r} \wedge m\vec{V}\right) \tag{0/24}$$

4. Conservation of energy:
   This equation, also known as the "first principle of thermodynamics", states that the total amount of energy within an isolated system must remain constant:

$$\frac{\mathrm{d}E_{\text{isolated system}}}{\mathrm{d}t} = 0 \tag{0/25}$$

5. Second principle of thermodynamics:
   By contrast with all four other principles, this is not a conservation statement. Instead this principle states that the total amount of entropy S within a constant-energy system can only increase as time passes; in other words, the entropy change of any system must be greater than the heat transfer Q divided by the temperature T at which it occurs:

$$\mathrm{d}S_{system} \geqslant \frac{\mathrm{d}Q}{T} \tag{0/26}$$

These are the only five important equations written in fluid mechanics. We usually apply these statements to our problem in either one of two ways:

- We may wish to calculate the *net* (overall) change in fluid properties over a zone of interest so as to relate them to the net effect (forces, moments, energy transfers) of the fluid flow through the zone's boundaries. This method is called **integral analysis** and we will develop it in chapter 3.
- We may instead wish to describe the flow through our zone of interest in an *extensive* manner, aiming to obtain vector fields for the velocity and pressure everywhere, at all times. This method is called **differential analysis** and we will develop it in chapter 4.

0.8 Classification of fluid flows

As we will see progressively, it is extremely difficult to obtain *general* solutions for fluid flow. Thus, whenever possible or reasonable, simplifying hypothesis are made about the behavior of any particular flow, that allow us to proceed with the analysis and obtain a reasonable, if inexact, specific solution. It is therefore a habit of fluid dynamicists to *classify* fluid flows in various categories, which are not necessarily incompatible. When approaching a given problem, we typically look out for the following characteristics:

Time dependence Flows which do not vary with time are called *steady*. Steadiness is dependent on the chosen point of view: for example, the air flow around an airliner in cruise flight is seen to be steady from within the aircraft, but is obviously highly unsteady from the point of view of the air particles directly in the path of the airliner. Steadiness is also dependent on the selection of a suitable time frame: time-variations in a flow may be negligible if the time window used in the study is short enough.

Mathematically, flows are steady when either the *partial time derivative* $\partial/\partial t$ or *total time derivative* $\mathrm{D}/\mathrm{D}t$ (concepts we will study in chapter 4) of properties is zero: this considerably eases the search for solutions. It is important not to confuse steadiness (property fields constant in time) with *uniformity* (properties constant in space).

To find out whether or not a flow is steady, compare instantaneous representations (photos, measurement readings, vector field representations, etc.) taken at different times. If they are all identical, the flow is steady. There does not exist an analytical method, however, that allows us to *predict* whether a flow will be steady.

Compressibility When the density of the fluid is uniform, the flow is said to be *incompressible*. The term is treacherous, because it refers to density, not pressure (incompressible flows almost always feature non-uniform pressure fields).

To find out whether a gas flow is compressible, compute the Mach number (eq. 0/10 p.15). Below 0,3 the flow is always incompressible. Compressibility effects can be reasonably neglected below [Ma] = 0,6. Unless very specific phenomena such as phase changes or extreme speeds occur, the flow of liquids is always incompressible.

Temperature distribution In a fluid flow, temperature changes can occur due to three phenomena:

- Heat transfer from solid bodies;
- Changes in pressure and density due to work being performed on the fluid (by moving solid bodies, or by the fluid itself);
- Heat created through internal friction within the fluid.

To find out whether a flow has uniform temperature (i.e. whether it is *isothermal*):

1. Quantify the heat transfer from external bodies. If that is zero, the flow is *adiabatic*, meaning there is no heat transfer.
2. Find out whether the flow is compressible. If not, the density changes are negligibly small and temperature changes do not occur due to compression or expansion.
3. Quantify the mechanical power lost by the fluid as it flows through the domain of interest. This power is integrally lost to heat.

Because the heat capacity of fluids is generally very high, temperature changes due to this third factor are usually negligibly small. This is assessed with an example in exercise 0.6 p.25.

Turbulence One last characteristic that we systematically attempt to identify in fluid flows is *turbulence* (or its opposite, *laminarity*). While laminar flows are generally very smooth and steady, turbulent flows feature multiple, chaotic velocity field variations in time and space.

We shall first approach the concept of turbulence in chapter 5, and study it more formally in chapter 6. In the meantime, we may assess whether a flow will become turbulent or not using a non-dimensional parameter named the *Reynolds number*, noted [Re]:

$$[\text{Re}] \equiv \frac{\rho V L}{\mu} \tag{0/27}$$

To find out whether a flow will become turbulent, quantify [Re] by using the fluid properties ρ and μ, a representative fluid speed V, and a length L which is representative of the flow domain (for example, the stream-wise length of an obstacle). If the result is on the order of 10^4 or more, the flow is very likely to become turbulent over the length L. By contrast, with [Re] of the order of 10^2 or less, the flow is very likely to remain laminar over this length.

This crude quantification has more meaning than it first appears. In particular, it assesses the influence of viscosity, which acts to to attenuate non-uniformities and unsteadiness in fluid flow, effectively "damping" its kinematics. We will explore the meaning behind this procedure, and apply it to concrete examples, in chapters 5 and following.

0.9 Limits of fluid mechanics

Fluid mechanics is a complex discipline.

It is easy to observe that flows as ordinary as sea waves crashing on a reef, water flowing down a river stream, or air blown into one's hands, display tremendous geometrical complexity. Even after choosing to describe only the movement of macroscopic fluid particles instead of individual molecules (and thereby avoiding studying thousands of billions of individual movements), we still need to describe a three-dimensional field of properties (pressure, temperature, etc.), one of which, velocity, is itself three-dimensional.

Thus, even before we begin describing the exact problem and a procedure to obtain its solution, we know that the mere description of a *solution* can have tremendous complexity.

Additionally, we have to admit that in practice much progress can be made in the field of fluid mechanics. For example, our weather forecasts have almost no value beyond one week, and aircraft manufacturers with budgets measured in billions of dollars still make extensive use of wind tunnel models – this despite our staggering continuous rate of progress in computing technology, and many decades of efforts dedicated to analytical fluid mechanics.

In our present study of fluid mechanics, therefore, we shall proceed modestly, and will always take care to show the limits of our analysis.